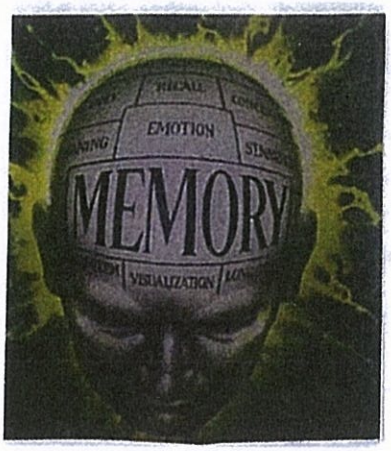
MATH & MEMORY



BARK'S DISCOVERY METHOD

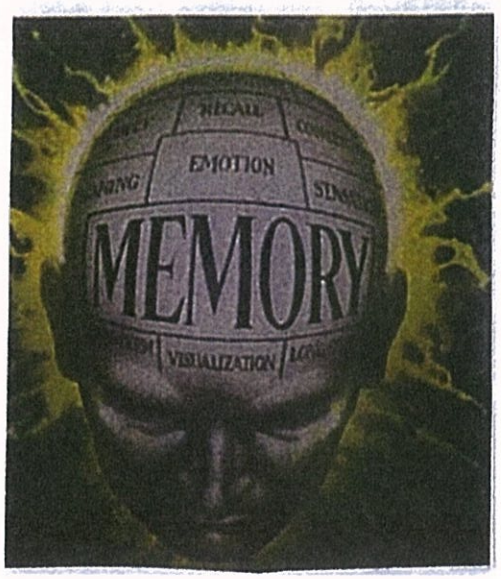
BOOK 4

FOR SECONDARY SCHOOLS

FREE RANGE LEARNING

THE 3RS THE PROFESSIONAL WAY

MATH & MEMORY



BARK'S DISCOVERY METHOD

BOOK 4

ISBN 949384 27 5

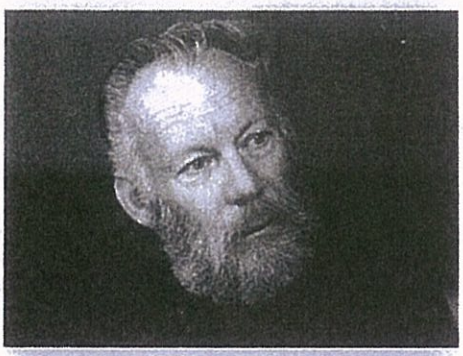
FREE RANGE LEARNING

THE 3RS THE PROFESSIONAL WAY

INDEX 4

THE DIRECT APPROACH

- Answers to questions are based on efficiency, the use of mental arithmetic and professional Memory Training Techniques.
- This approach satisfies the following statement:
  Minister for Education and Training
  I refer to your letter concerning the mathematics education of students in New South Wales. As you have acknowledged, the Board of Studies **does not prescribe particular teaching methods** for its curricula. Teachers use their professional judgement to select appropriate approaches for different situations involving individual students or groups of students.
- The archaic "Necessary Working" request stems from a time that the rigid classical way was only for the gifted. Consequently, teachers must not intimidate or discourage students who use my Discovery Method!



Author's Background

Born 20.12.1928

1.	**H.S.C. (HOLLAND)** 4 Unit Maths, Mechanics, Technical Drawing Physics, Chemistry, Biology, Political Economy History, Geography, Dutch, English. French German, Art, P.E.		1947
2.	Certificate of Ability, Nautical College	Holland.	1949
3.	Diploma 3rd. Mate, Sea Going Trade	Holland	1951
4.	Diploma 2nd. Mate, Sea Going Trade	Holland	1954
5.	Spanish Commercial Correspondence	Holland	1954
6.	French Commercial Correspondence	Holland	1958
7.	English Commercial Correspondence	Holland	1961
8.	Language Studies: Friesian, Italian, B.A. French		
9.	High School Teacher: English & French	Holland	2 yrs
10.	High School Teacher	Australia	14 yrs

De La Salle, Ashfield
Latin, French & English.

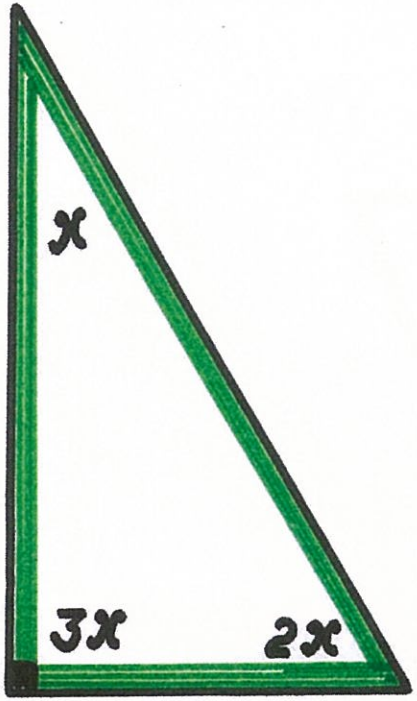
St. Dominic's, Kingswood
Creative Writing, English, Subject Master Technical Drawing

Patrician Brothers, Granville
Creative Writing, Mathematics, Subject Master Music

Oakhill College, Castle Hill
Creative Writing, Mathematics, Subject Master Technical Drawing, French & Art

11.	Insurance & Real Estate Agent (Finance)	
12.	Owner Builder (Rammed Earth)	
13.	Hawkesbury Adult Education	Creative Writing, Spanish.
14.	Professional Musician	Accordion, Flamenco Guitar.
15.	Author of Textbooks	English & Mathematics
16.	Private Tutor since 1976:	K-12
17.	Soccer Coach	

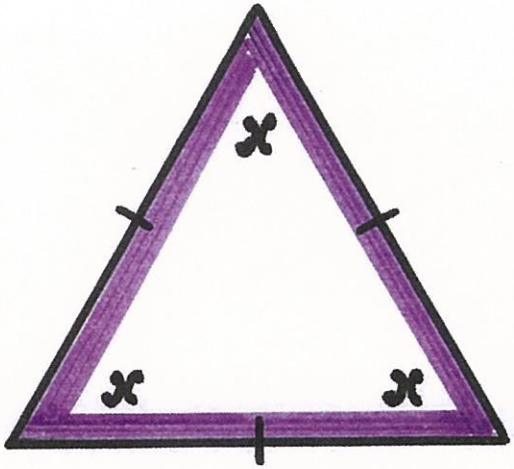
THE 3RS
THE PROFESSIONAL WAY
A. BARK, CATTAI. N.S.W. 2756



RIGHT TRIANGLE

$6x = 180^\circ$

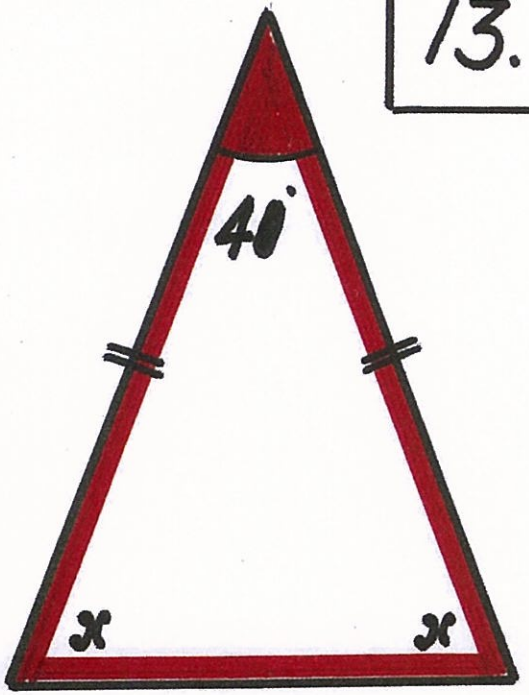
$x = 30^\circ$



EQUI LATERAL

$3x = 180^\circ$

$x = 60^\circ$



ISO SCELES

$2x = 140^\circ$

$x = 70$

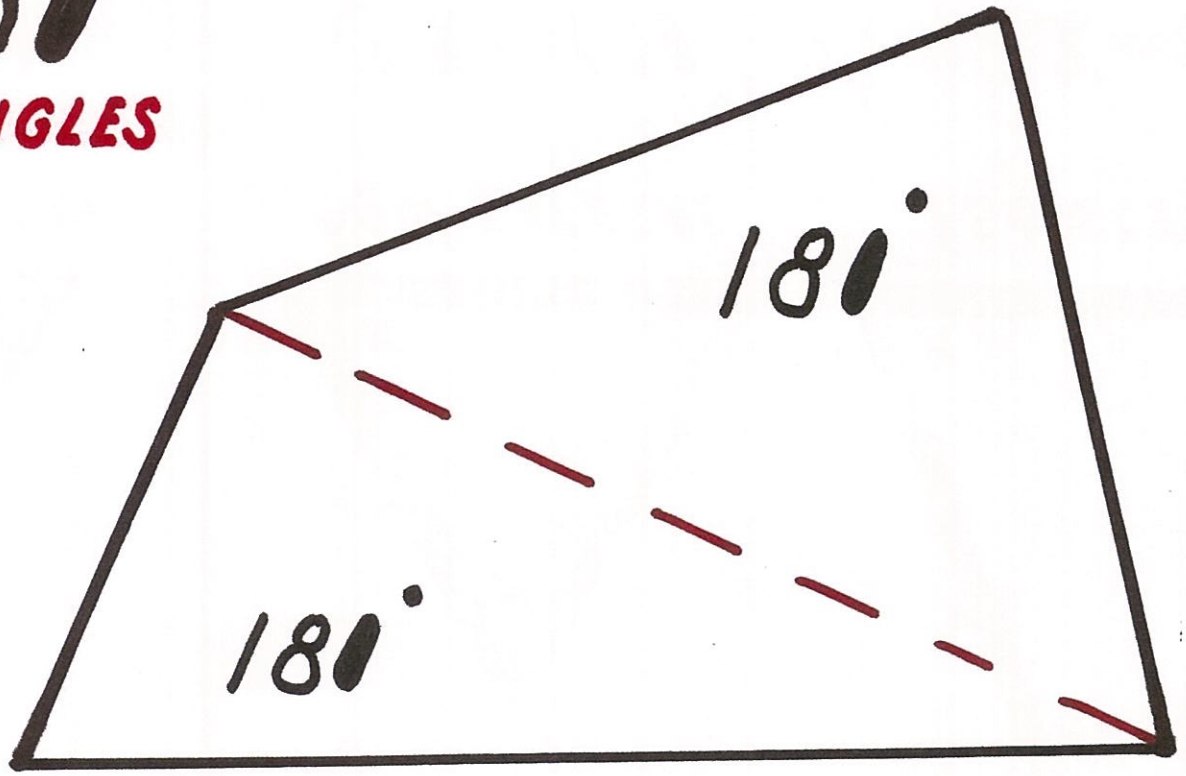
ANGLE SUM QUADRILATERAL

FOUR SIDES

14.

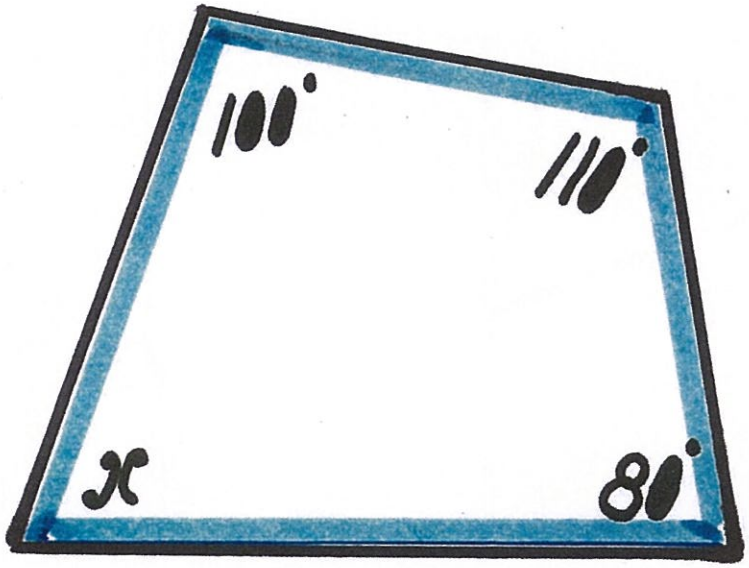
360°

2 TRIANGLES



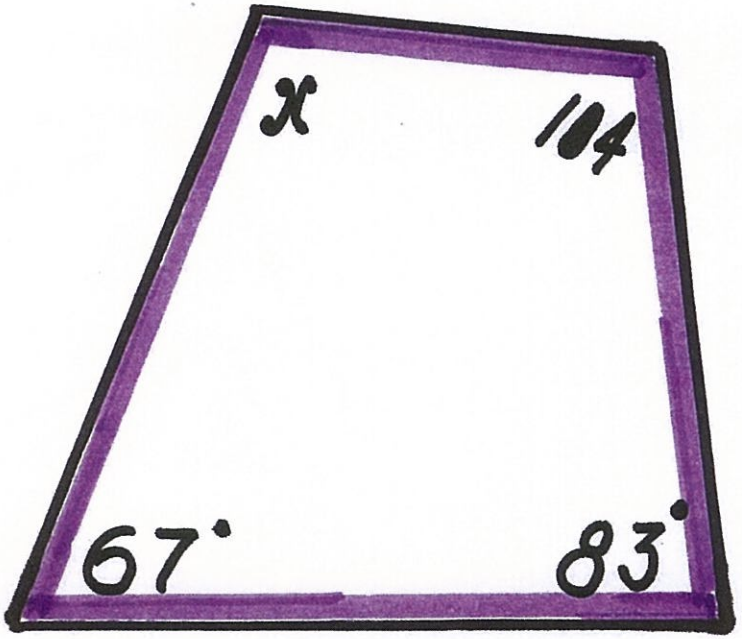
15.

MENTALLY



$x = 70^\circ$

BY CALCULATOR



START WRITE

$x = 360 - 104 - 83 - 67 = 106^\circ$

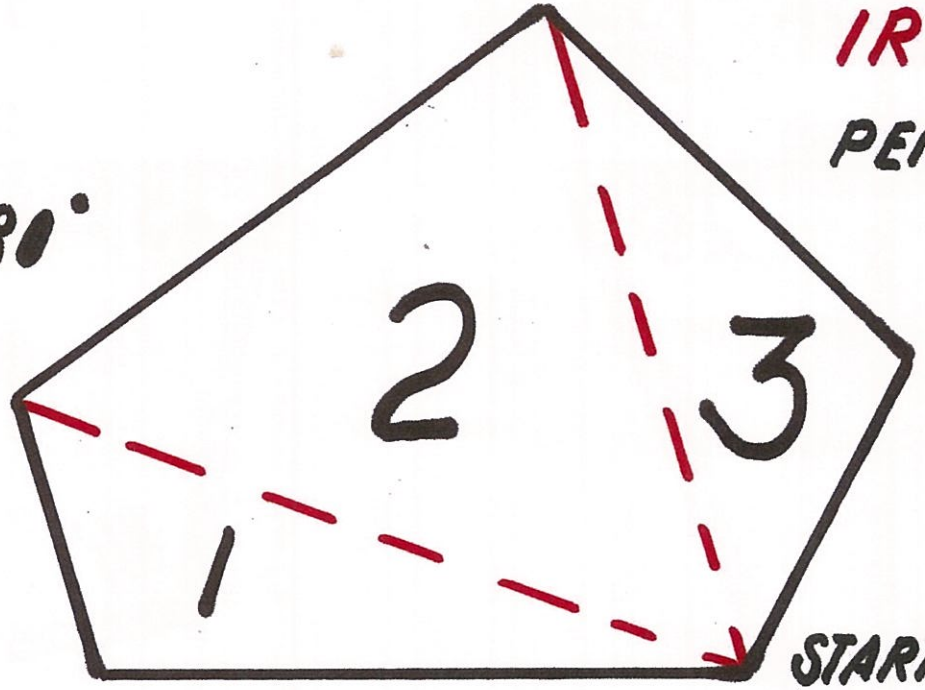
ANGLE SUMS POLY (MANY) GONS (ANGLES)

5 SIDES

5 VERTICES

$5-2=3\times 180°$

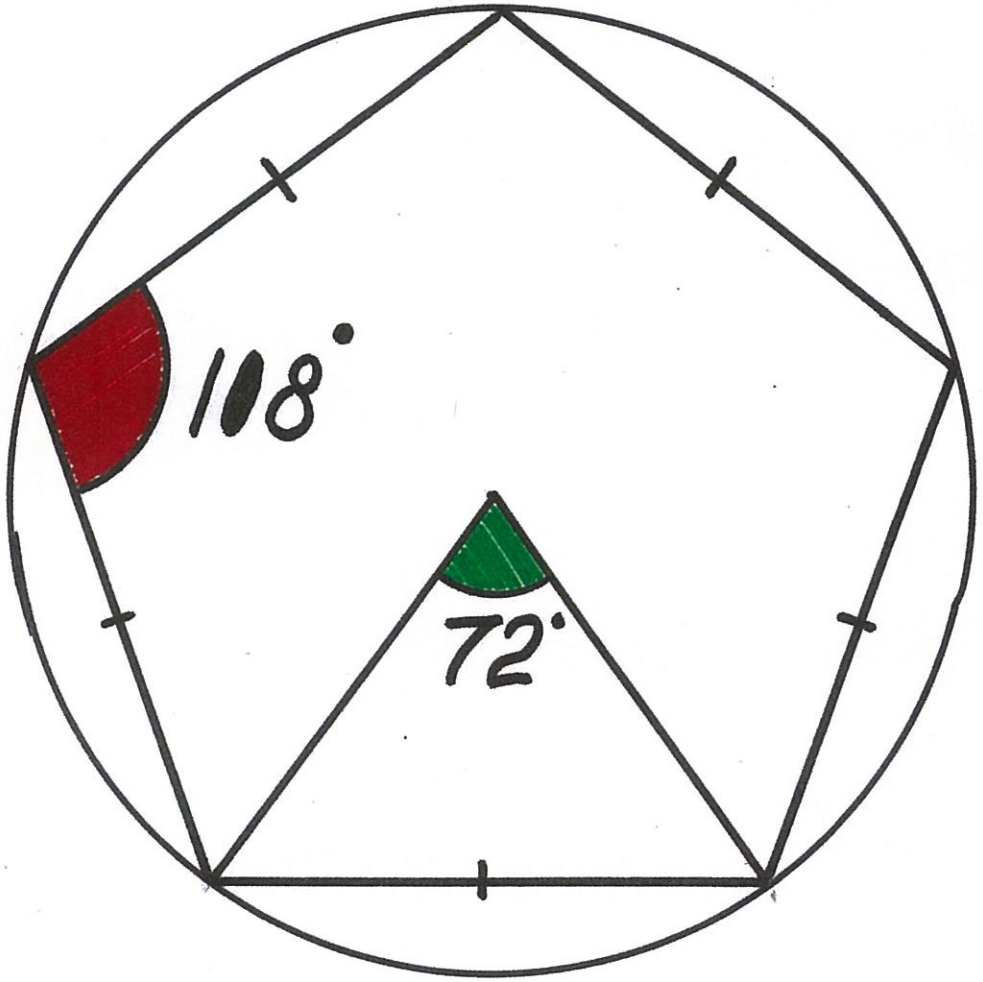
3 TRIANGLES



IRREGULAR PENTA GON

IN GENERAL	n SIDES $\rightarrow (n-2)$ TRIANGLES

REGULAR PENTAGON



5

INTERIOR ANGLES

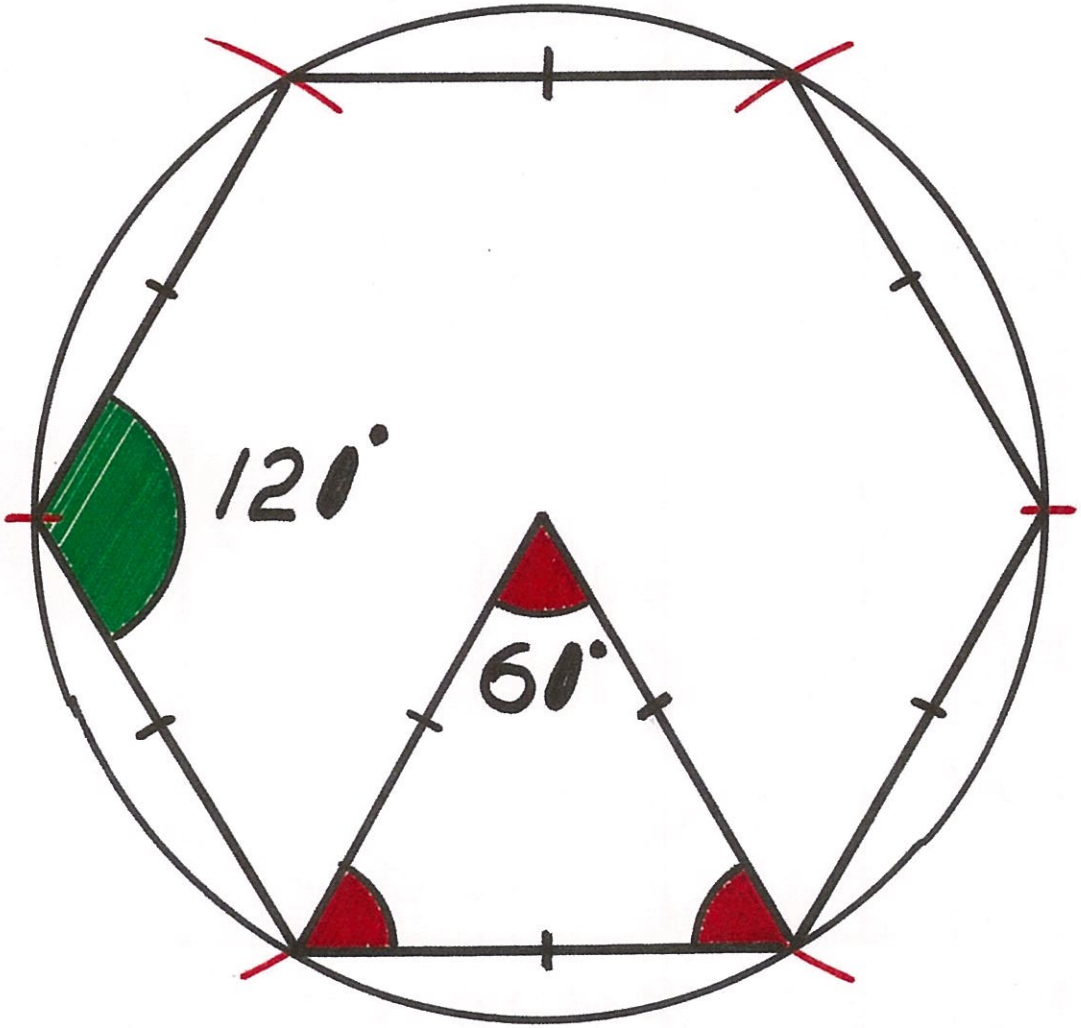
$(3 \times 180^\circ) \div 5 = 108^\circ$

CENTRE ANGLES

$360^\circ \div 5 = 72^\circ$

REGULAR HEXAGON

18.



a

SOME OTHER POLYGONS

HEPTAGON
SEVEN

REGULAR

INTERIOR ANGLE SUM

$6 \times 180^\circ = 1080^\circ$

135° EACH

CENTRE ANGLES 45°

NONAGON
NINE

REGULAR

INTERIOR ANGLE SUM

$8 \times 180^\circ = 1440^\circ$

144° EACH

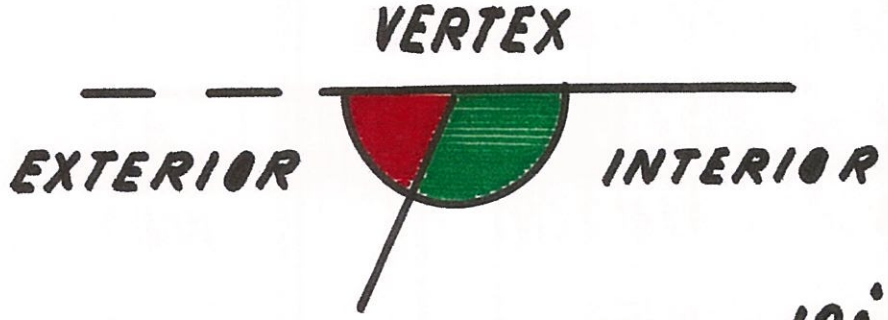
CENTRE ANGLES 36°

OCTAGON
EIGHT

REGULAR

DECAGON
TEN

EXTERIOR ANGLE SUM ANY POLYGON



SUM AT ONE VERTEX = 180° = 1 TRIANGLE

FOR n VERTICES: IN + EX = n TRIANGLES

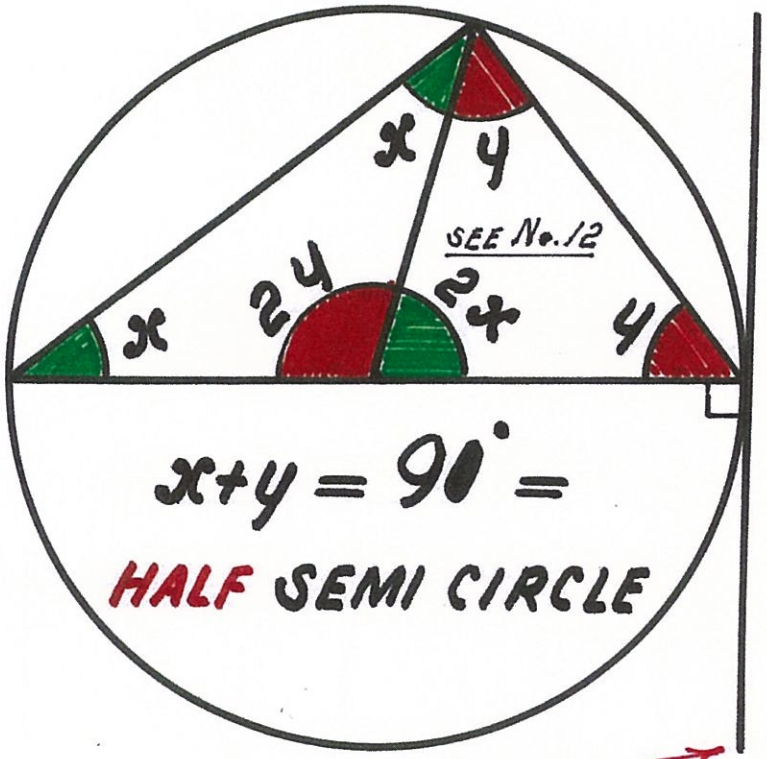
IN = (n-2) —"—

EX = 2 —"—

= 360°

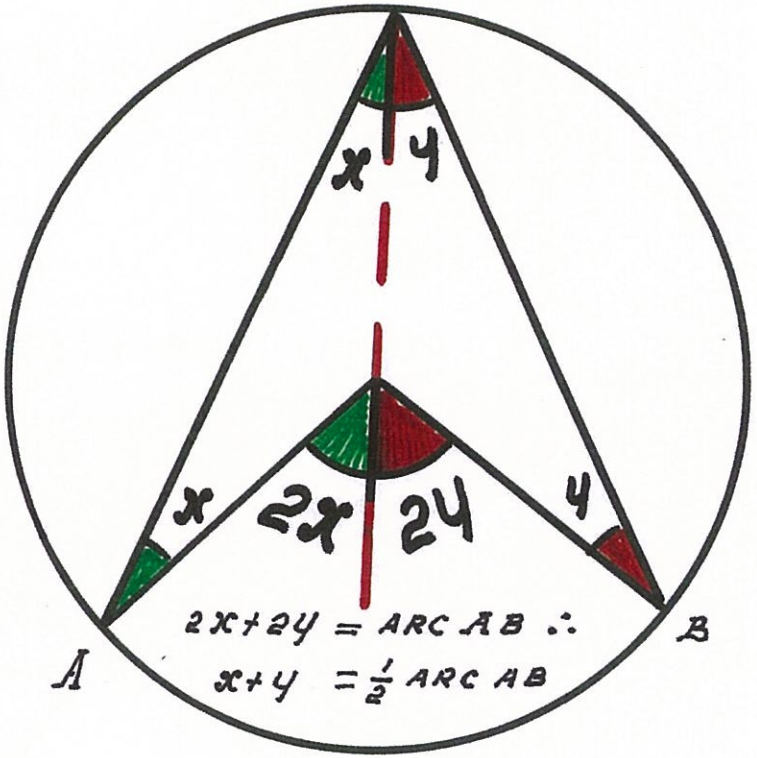
CIRCUMFERENCE ANGLES ARE HALF CENTRE ANGLES
STANDING ON THE SAME ARC

21.



$x+y = 90° =$

HALF SEMI CIRCLE



$2x+2y$ = ARC AB ∴

$x+y = \frac{1}{2}$ ARC AB

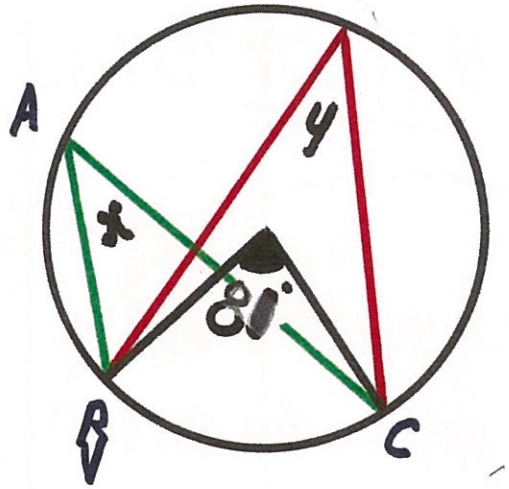
TANGENT ⊥ RADIUS

PERPENDICULAR TO

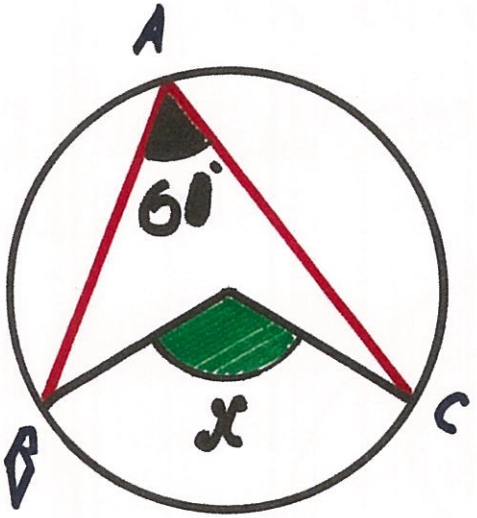
EXTERIOR = SUM REMOTE

EXTERIOR = SUM REMOTE

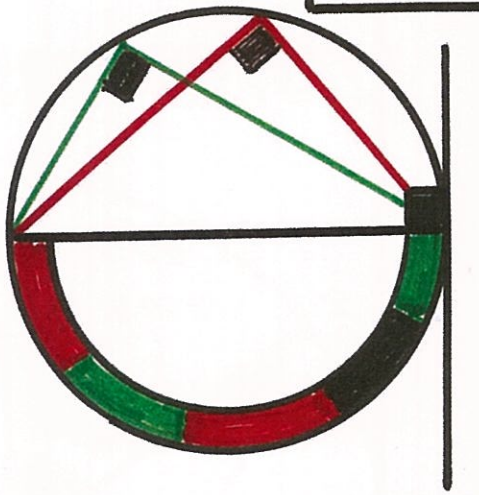
22.



$x = y = 40^\circ$



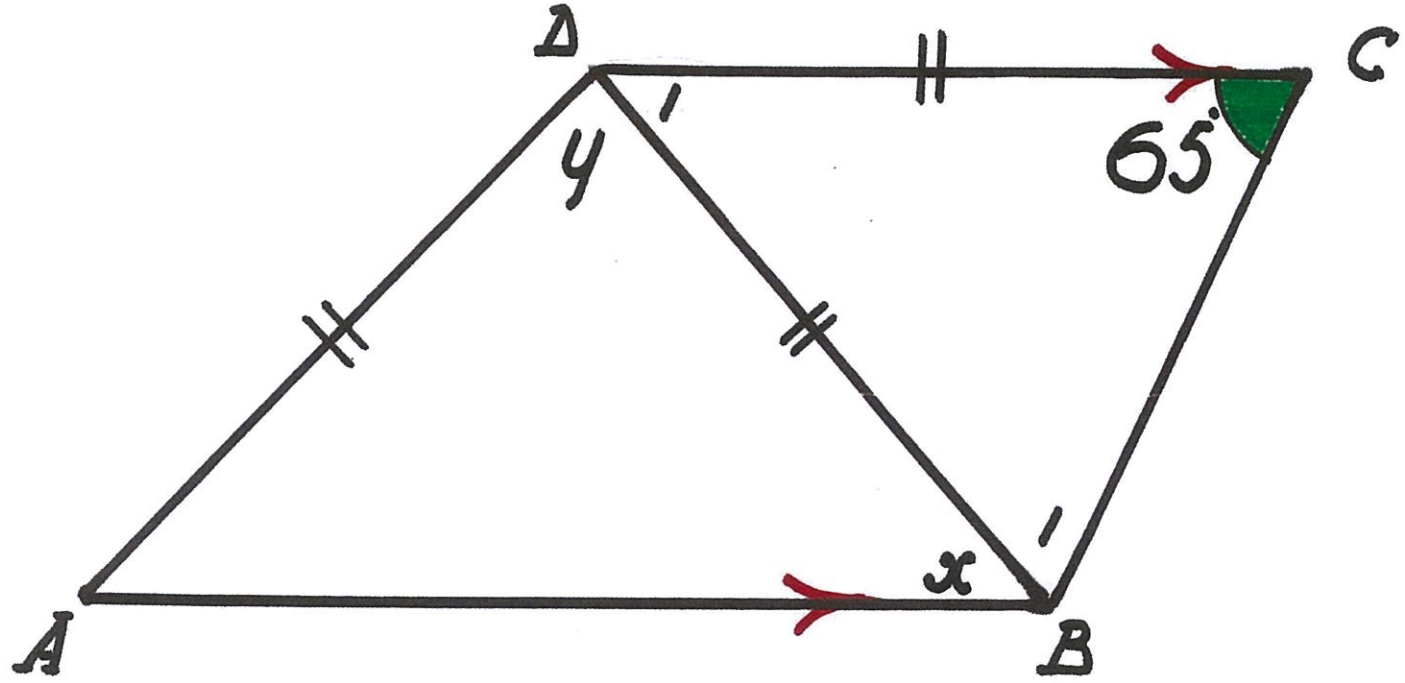
$x = 120^\circ$



RIGHT ANGLES ON SEMI CIRCLE

01

23.



$\hat{B}_1 = \hat{C} = 65^\circ \therefore \hat{D}_1 = x = 50^\circ (ALT) = \hat{A} \therefore y = 80^\circ$

ANGLE RATIO TRIANGLE

1 : 3 : 5

THINK 9 PARTS = 180°

ANGLES ARE 20° 60° 100°

ANGLES: TASK

1 RIGHT TRIANGLE
ANGLES: $6x = 90^\circ$, $4x =$

EQUILATERAL
ANGLES $2x$

ISOSCELES
BASE ANGLES 50°

2 ANGLE SUM QUADRILATERAL

3 ANGLES x, 70°, 120°, 80°

4 MANY ANGLE
POLIGON: 7 SIDES. ANGLE SUM

5 REGULAR PENTAGON ANGLE
CENTRE- & INTERIOR ANGLES

6 REGULAR HEXAGON ANGLE
CENTRE & INTERIOR ANGLES

7 REGULAR OCTAGON
ANGLES

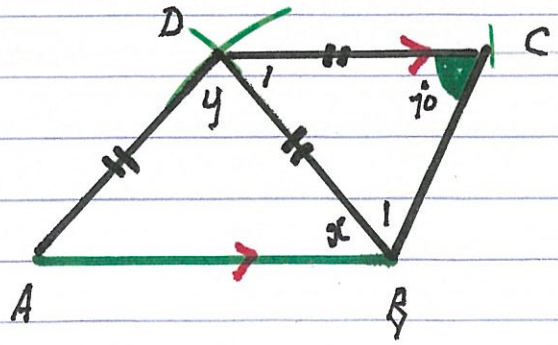
REGULAR NONAGON
ANGLES

8 EXTERIOR ANGLE SUM ANY POLYGON

10 CENTRE ANGLE 100°
$x + y$

AROUND FERRY
CIRCUMFERENCE.
ANGLE 70°, CENTRE ANGLE x

11



ANGLE RATIO

TRIANGLE

1 : 2 : 3

APPROXIMATIONS

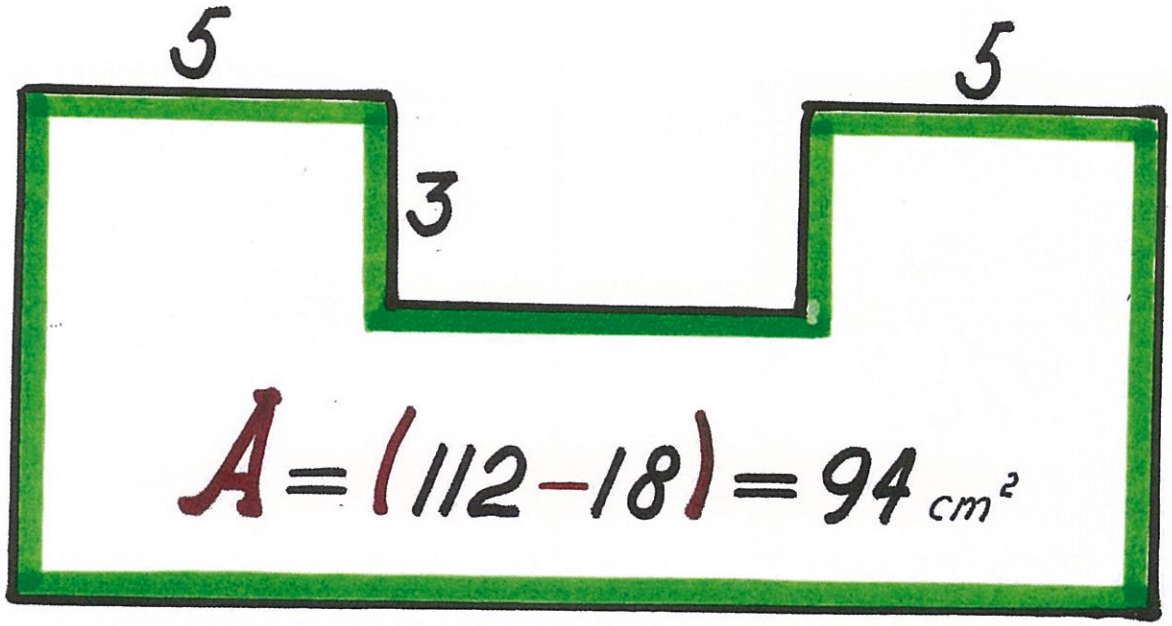
ALL MEASUREMENTS ARE APPROXIMATIONS
DUE TO
HUMAN ERROR, LIMITATIONS OR FAULTS OF INSTRUMENT

ROUNDING OFF

1.	16 NEAREST TEN	20
2.	12.4 NEAREST WHOLE	12
3.	12.5 NEAREST WHOLE	13
4.	249 NEAREST HUNDRED	200
5.	1.265 (↑ ↑ ↑) FROM 3 TO 2 DECIMAL PLACES TO 1 D.P.	1.27 ... 1.3
6.	15.372 FROM 5 TO 4 SIGNIFICANT FIGURES	15.37
7.	7230 FROM 4 TO 1 S.F.	7000
8.	ESTIMATE: 9.8×2.1	20 12 (12.5)
9.	PRACTICAL SITUATIONS HOW MANY $2.69 BOOKS FOR $500	$500 \div 2.69 = 185$
10	HOW MANY ZIPPERS TO SEW IN @ 90 CENTS TO EARN $250	$250 \div .9 = 278$ (finish the last zipper!)

COMPOSITE SHAPES

AROUND MEASURE

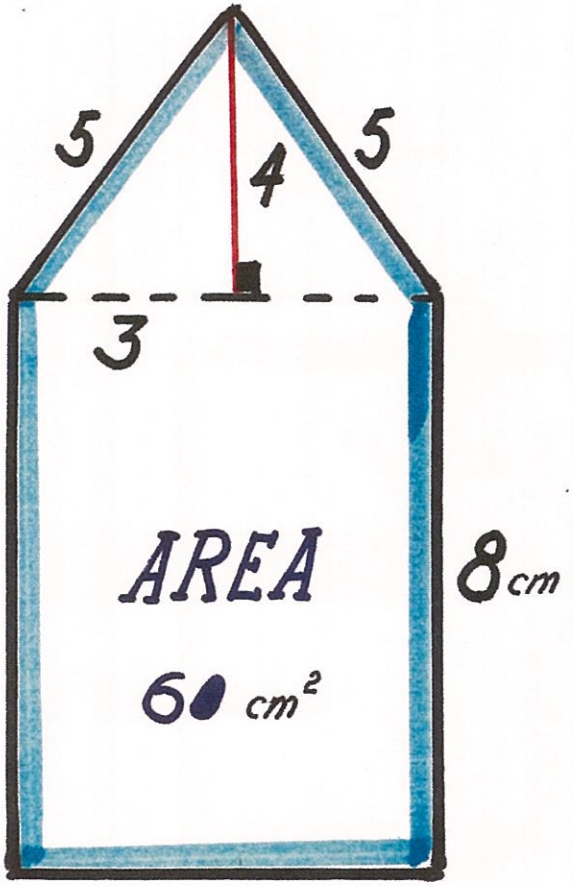


$A = (112 - 18) = 94\ cm^2$

16 cm

$P = 52$ cm

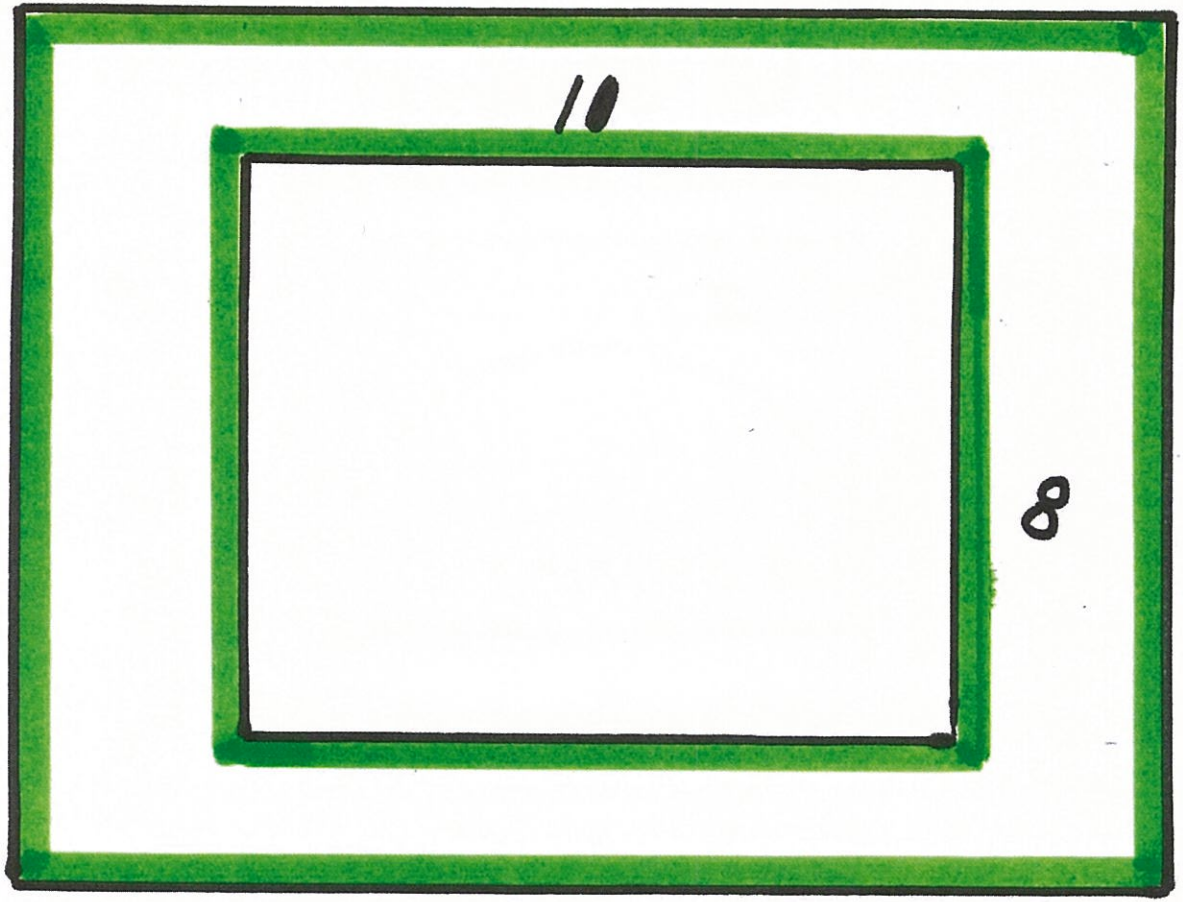
9/



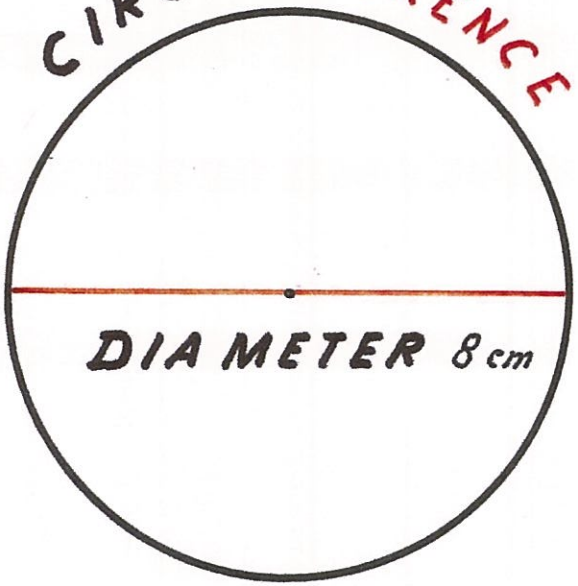
$P = 32$ cm

51.

$P = 40$ cm



BORDER AREA 112 cm^2



$C = \pi D = 2R\pi$ cm

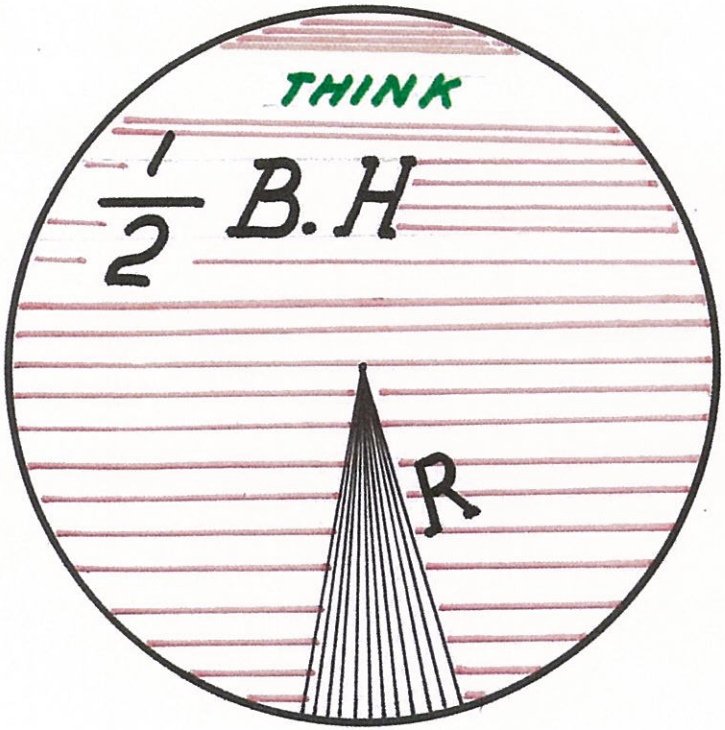
D D D

THE CIRCUM FERENCE – SPREAD OUT –

IS A BIT MORE THAN 3 TIMES THE DIAMETER.

CALLED PI (π) – P FOR PERIMETER – $\doteqdot \frac{22}{7} \doteqdot 3.14$

GREEK LETTER



A CIRCLE IS THE SUM OF AN INFINITE NUMBER OF TRIANGLES TOTAL BASE $2R\pi$ HEIGHT R ∴

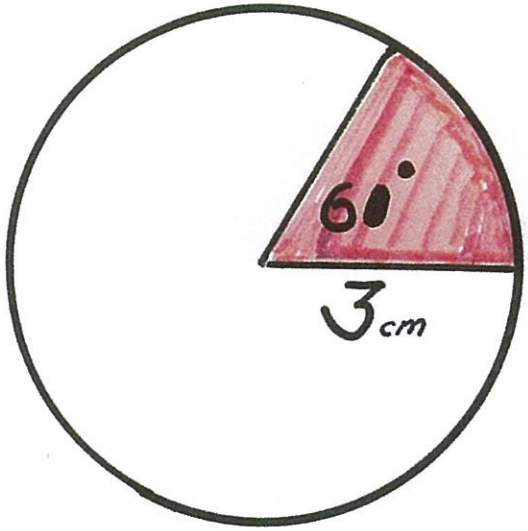
$$\text{AREA} = R^2\pi \ u^2$$

APPLY FORMULAS MENTALLY

54.

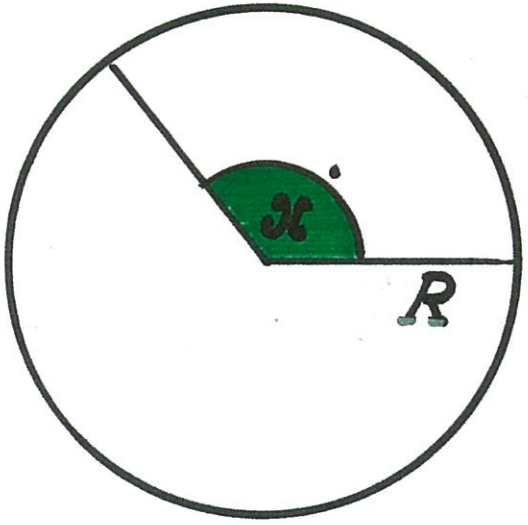
EXPRESS ANSWERS IN π FIRST

R	3	4	5	6
C THINK DOUBLER	6π	8π	10π	12π
A THINK SQUARER	9π	16π	25π	36π



AREA SECTOR $\frac{1}{6} \times 9\pi = \frac{3\pi}{2}$ cm²

PERIMETER $\frac{1}{6} \times 6\pi + 6 = (\pi + 6)$ cm



IN GENERAL:

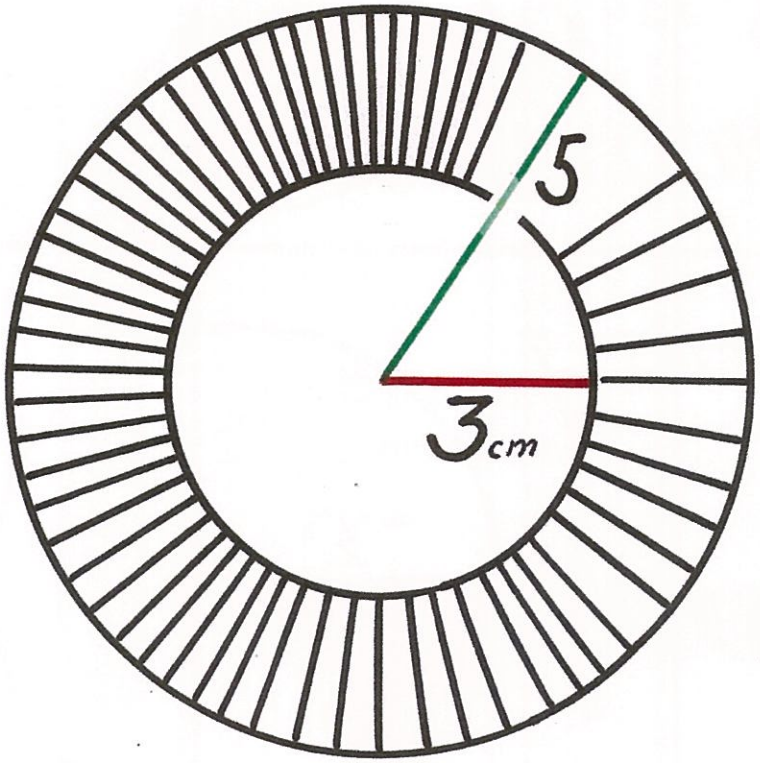
AREA SECTOR $= \frac{x}{360} \times R^2\pi$

PERIMETER $= \frac{x}{360} \times 2R\pi + 2R$

ANNULUS

56.

RING: COMPARE ANNUM - YEARLY CYCLE



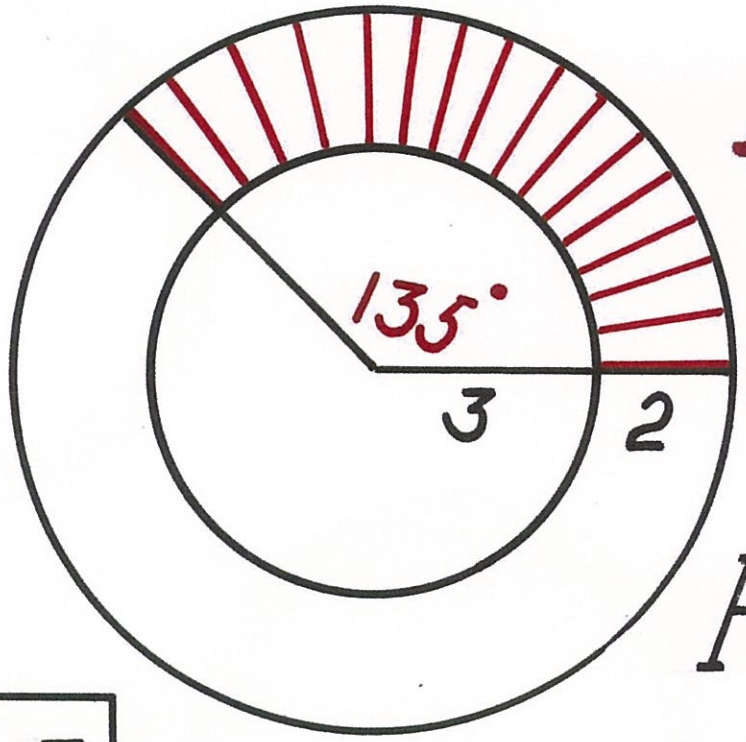
AREA: 16π cm²

TOTAL PERIMETER: 16π cm

22

PART ANNULUS

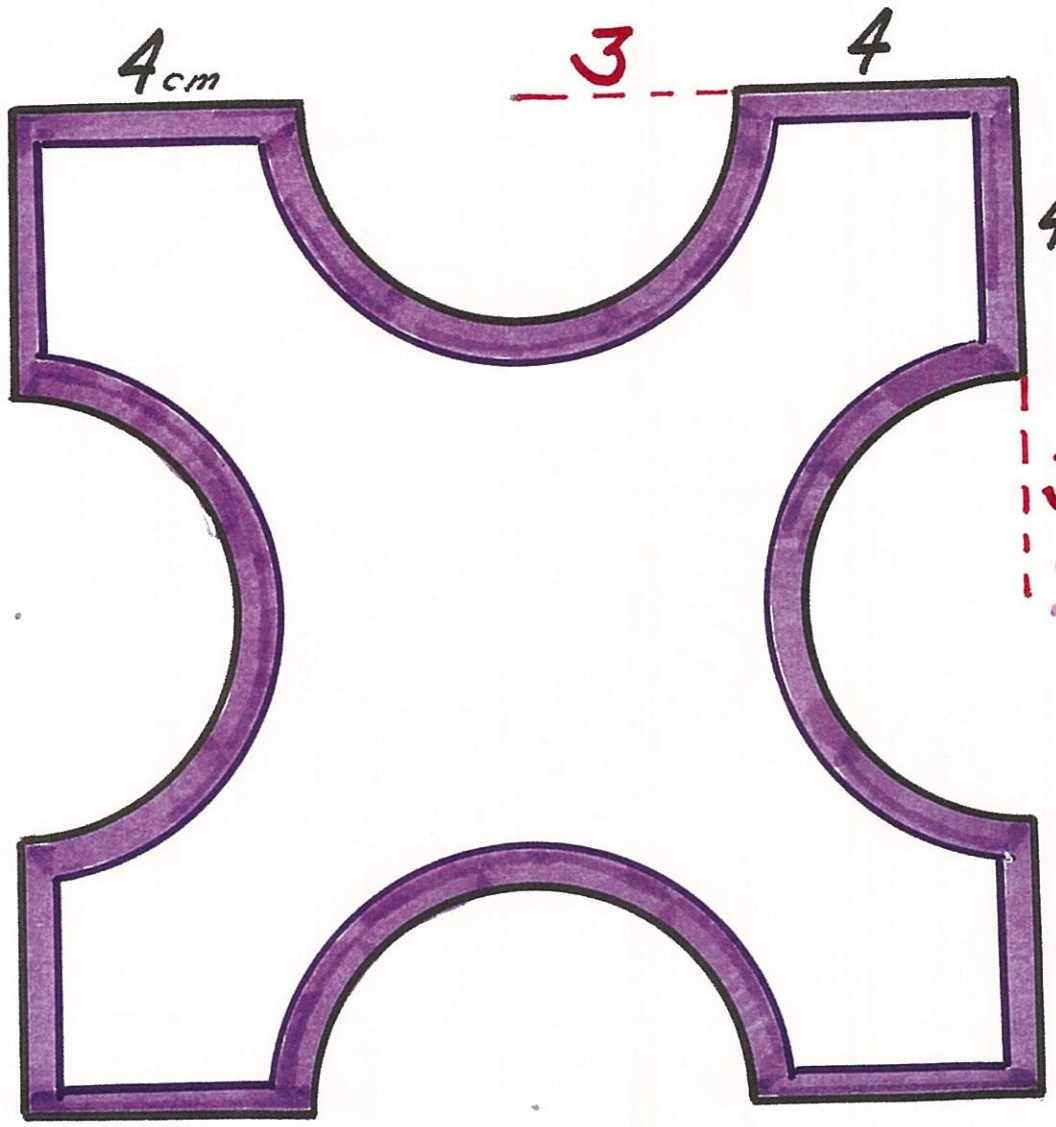
57.



AREA: $\frac{3}{8} \times 16\pi = 6\pi \text{ cm}^2$

PERIMETER: $(6\pi + 4) \text{ cm}$

$\frac{135}{360} = \frac{3}{8}$



12

AREA

$(196-18\pi)\ cm^2$

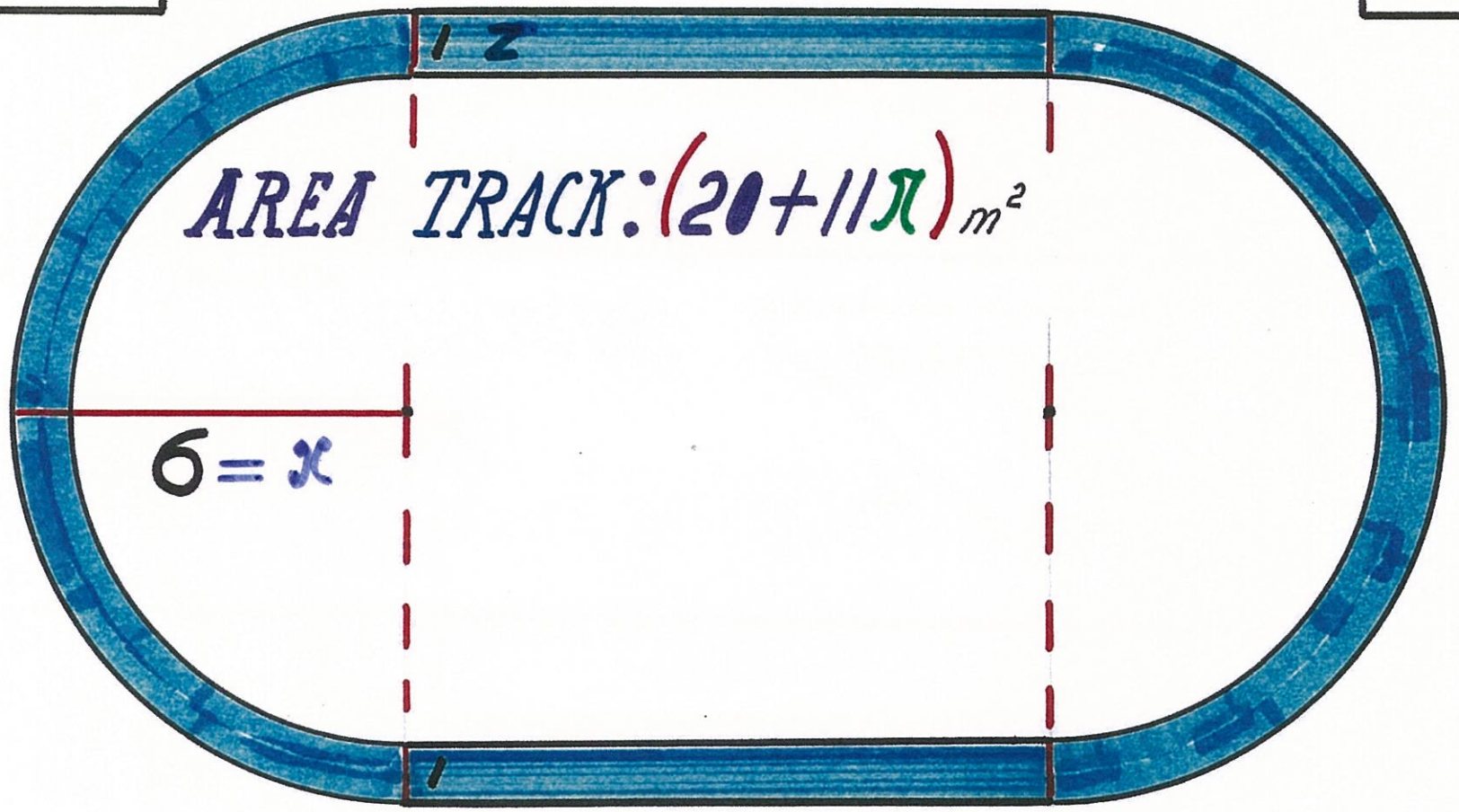
PERIMETER

$(32+12\pi)\ cm$

SCALE 1:100

$10_m = 4$

59.



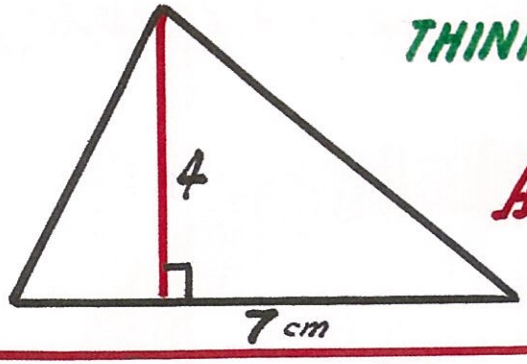
AREA TRACK: $(20 + 11\pi)_{m^2}$

TOTAL PERIMETER: $(40 + 22\pi)_m$

25

61.

1.



THINK $\frac{1}{2}$B.H

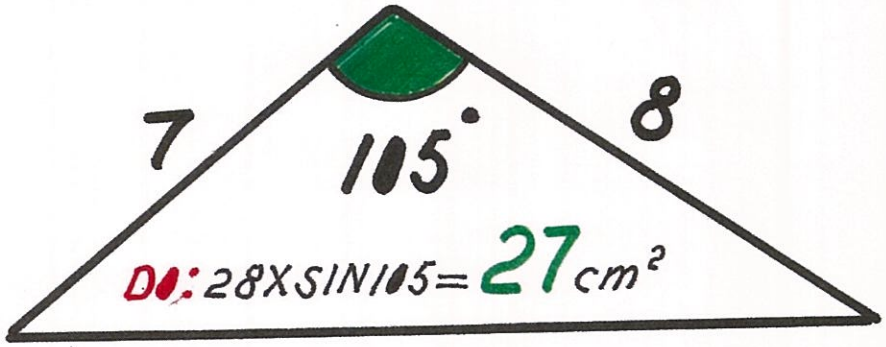
AREA: 14 cm²

2.

THINK

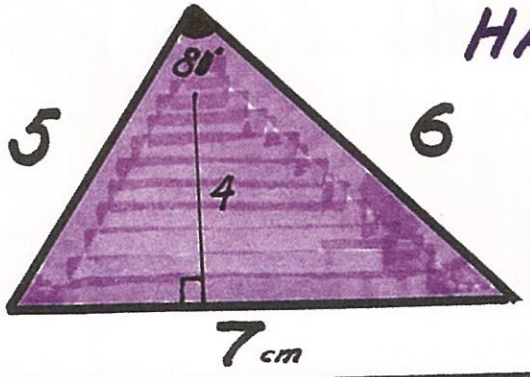
AREA: $\frac{1}{2}$ X 7X8XSIN105

LOOKS LIKE HALF B.H.



DO: 28XSIN105 = 27 cm²

3.



HALF SUM SIDES = 9

AREA = $\sqrt{9 \times 4 \times 3 \times 2}$ (9-5 | 9-6 | 9-7)

APPROXIMATION

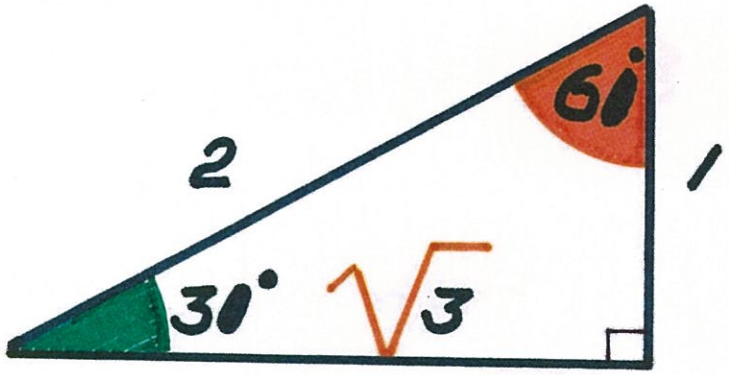
$\doteqdot$ 14.7 cm²

	METHOD
14 cm²	1.
14.8 cm²	2.
$\doteqdot$ 14.7 cm²	3.

TRIGONOMETRY: EXACT VALUES

613.

TRI ANGLE MEASURE

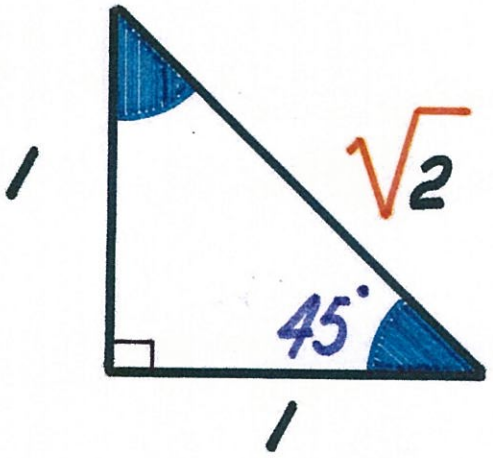


$\sin 30^\circ = \cos 60^\circ = .5$

$\sin 60^\circ = \cos 30^\circ = \sqrt{3}/2$

$\tan 30^\circ = \cot 60^\circ = 1/\sqrt{3}$

$\tan 60^\circ = \cot 30^\circ = \sqrt{3}$



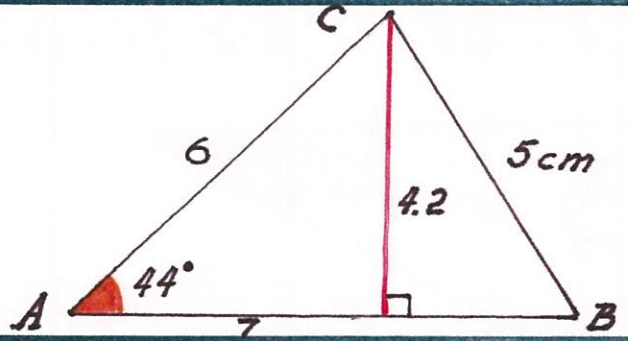
$\sin 45^\circ = \cos 45^\circ = 1/\sqrt{2}$

$\tan 45^\circ = \cot 45^\circ = 1$

TRIANGLE AREAS

$a=5,\ b=6,\ c=7,\ S=\frac{1}{2}(5+6+7)=9$

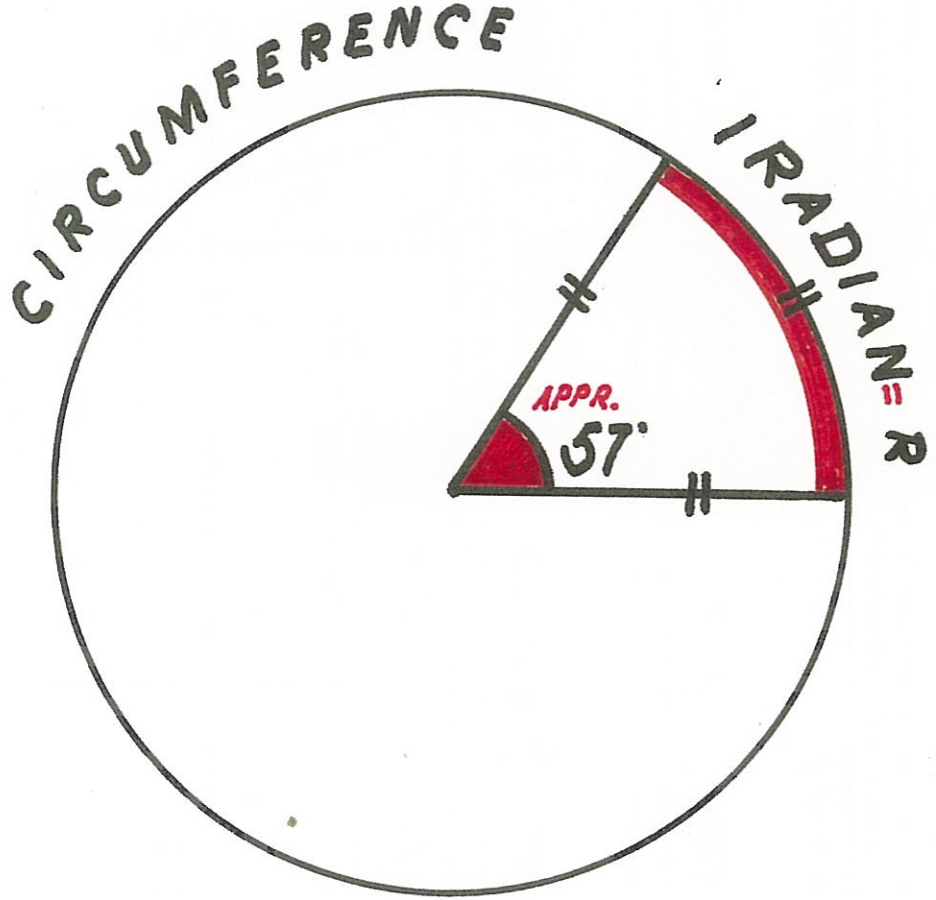
$A=\sqrt{S(S-5)(S-6)(S-7)}=14.7\ cm^2$



$A = 2.1 \times 7 = 14.7\ cm^2$

$A = 21 \times SIN\ 44 = 14.6\ cm^2$

LINEAR MEASURE IN RADIANS ≡ RADII CASIO MODE 5



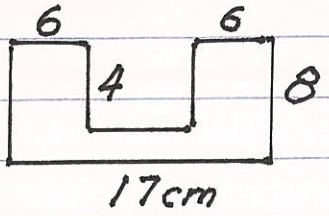
$\pi^c = 180°$

1 RAD ≒ 57°

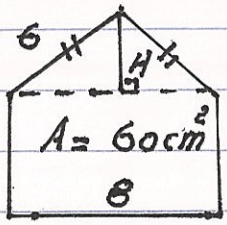
CIRCUMFERENCE $= 2\pi R = 2\pi$ RAD OR $2\pi^c$ OR $2\pi = 360°$

AREA: TASK

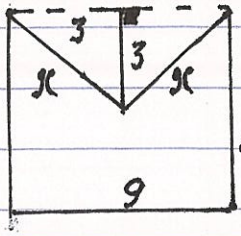
15 A & P



16 A & P & H

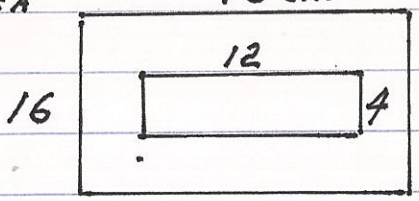


16 A & P



BORDER AREA

17



18 **19** CIRCLE $D = 10\,cm$

$C = \pi D = 2R\pi$

$A = R^2\pi$ PLACE CHANGE TO REMEMBER

21 SECTOR $40°$, $R = 4\,cm$ A & P

SECTOR $120°$, $R = 6\,cm$ A & P

22 ANNULUS

R 6 & 4 cm A & P

23 PART ANNULUS

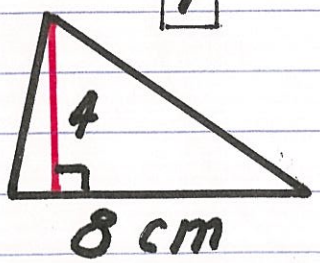
R 6 & 4 cm, $X = 120°$

24 $X = 6\,cm$, $Y = 5\,cm$

25 WIDTH

$X = 8$, $Y = 12$, $Z = 2\,cm$

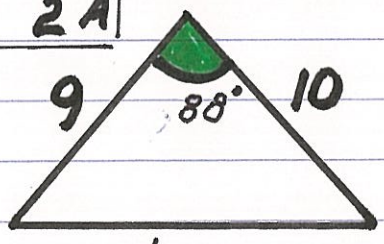
26 **1**



A =

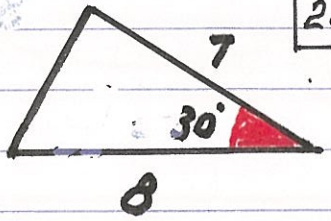
SEE EXACT RATIOS PAGE 27. BUT NOT BY CALCULATOR!

2A



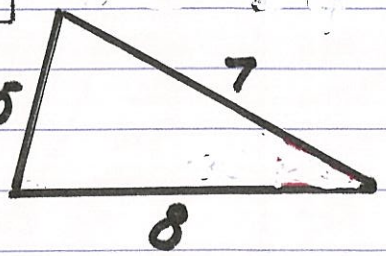
A =

28



A =

3



A =

SEE PAGE 28.

ANGLES

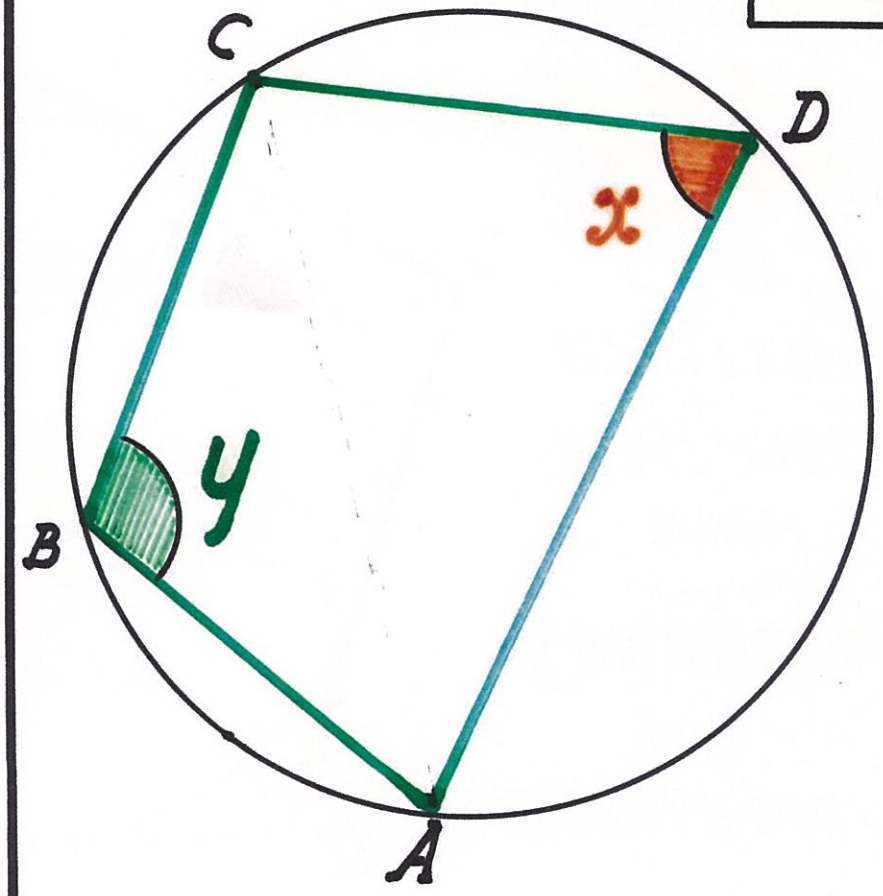
$x, 2x, 3x, 4x$

$10x = 360^\circ$

$x = 36^\circ$

CYCLIC QUADRILATERAL

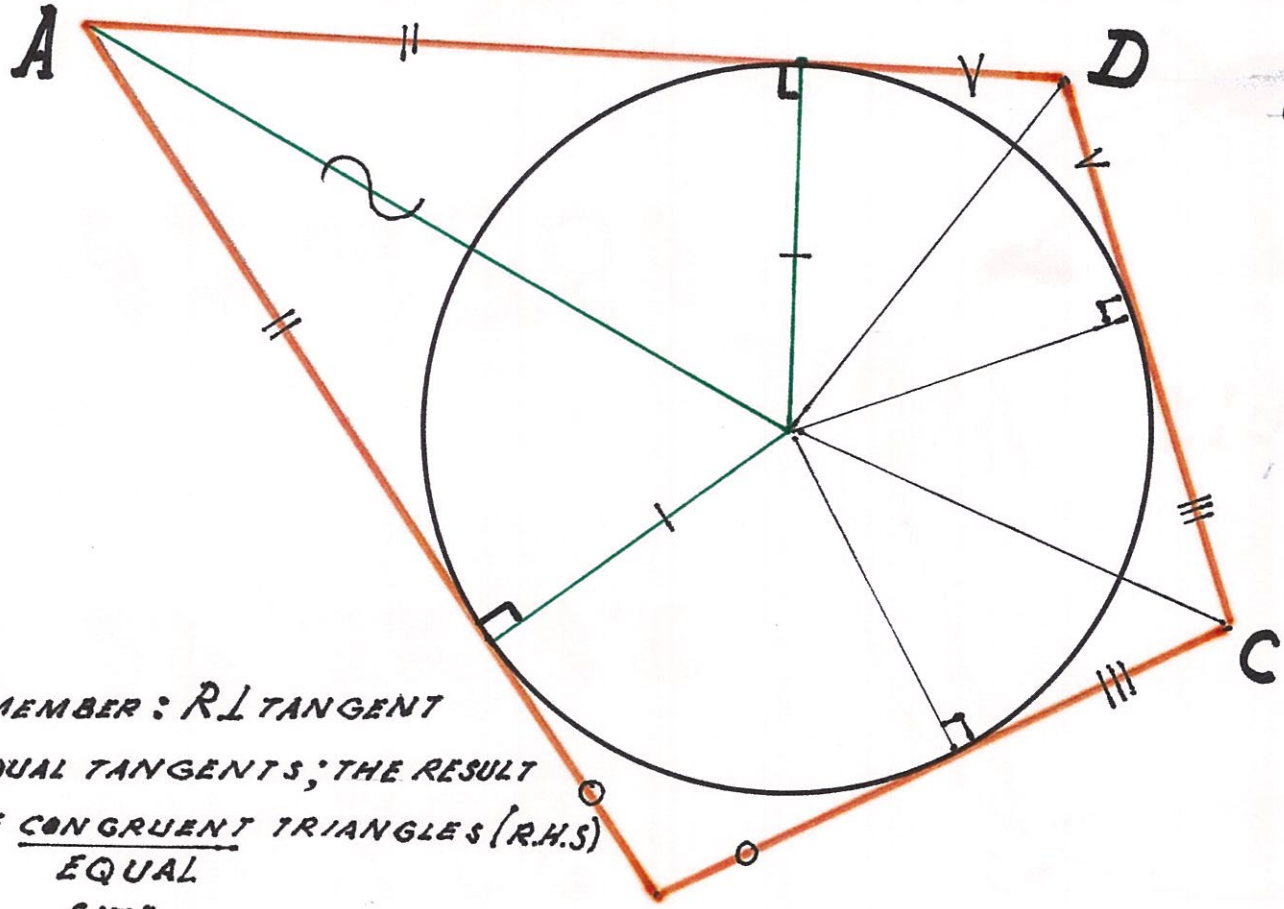
915.



$x + y = \frac{1}{2}\,\text{ARC}\,ABC + \frac{1}{2}\,ADC = 180^\circ$

SUM OPPOSITE SIDES

9/6.



2 TRIANGLES ARE CONGRUENT (EQUAL) WHEN RIGHT ANGLE & HYPOTENUSE & 1 SIDE ARE EQUAL

(R H S)

REMEMBER : R⊥ TANGENT EQUAL TANGENTS, THE RESULT OF CONGRUENT (EQUAL SIZE) TRIANGLES (R.H.S)

$$AD + BC = AB + DC$$

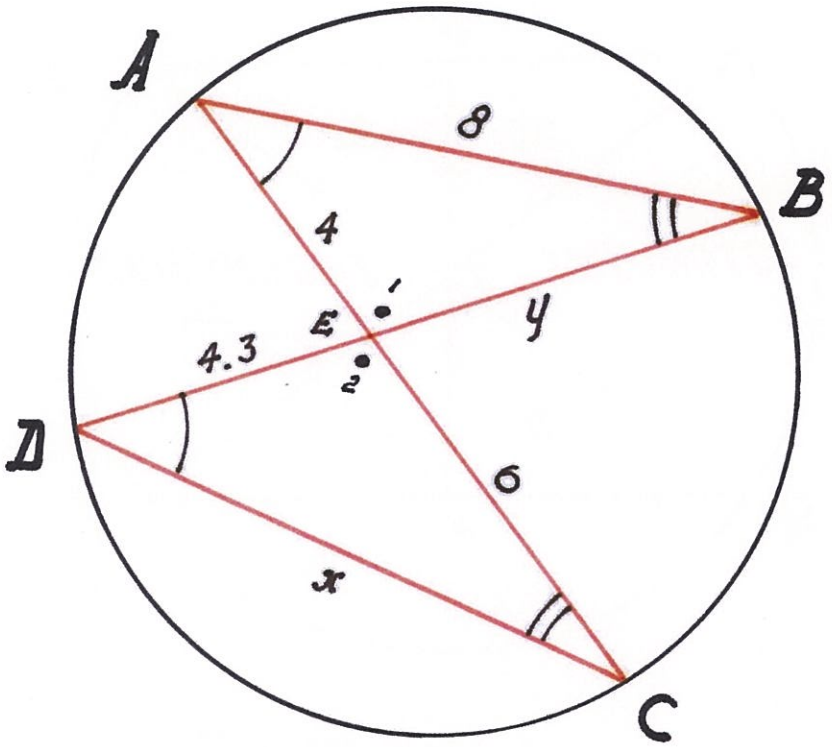
CYCLIC QUADRILATERAL

33

$\hat{A} = \hat{D}$ (ON ARC BC). $\hat{B} = \hat{C}$ (ON ARC AD). $\hat{E}_1 = \hat{E}_2$ (OPP.) $\therefore$

$\triangle ABE$ IS SIMILAR TO $\triangle DCE$ EQUIANGULAR

SAME SHAPE, DIFFERENT SIZE



BOTH OPPOSITE

$$\frac{4}{4.3} = \frac{y}{6} = 5.6$$

BOTH OPPOSITE

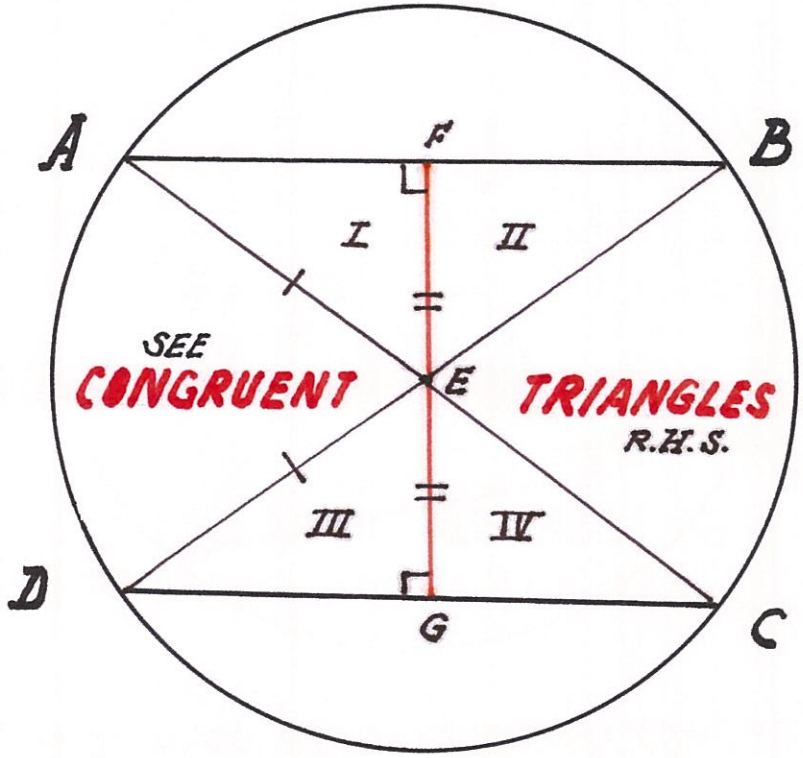
TIMES

DIVIDE

$$\frac{4}{4.3} = \frac{8}{x} = 8.6$$

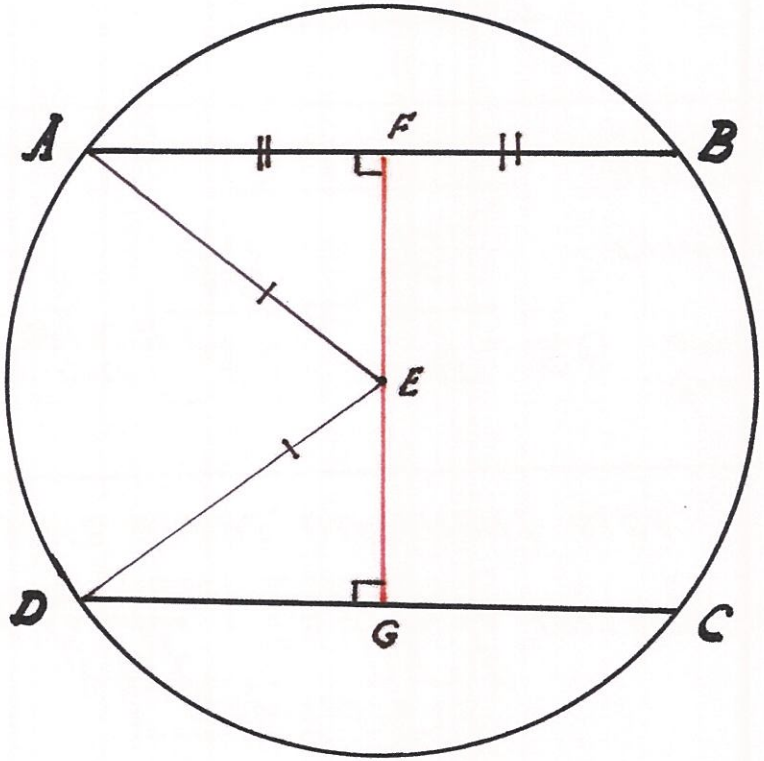
BOTH OPPOSITE

$EF = EG$



CHORDS EQUIDISTANT FROM THE CENTRE ARE EQUAL & HALVED

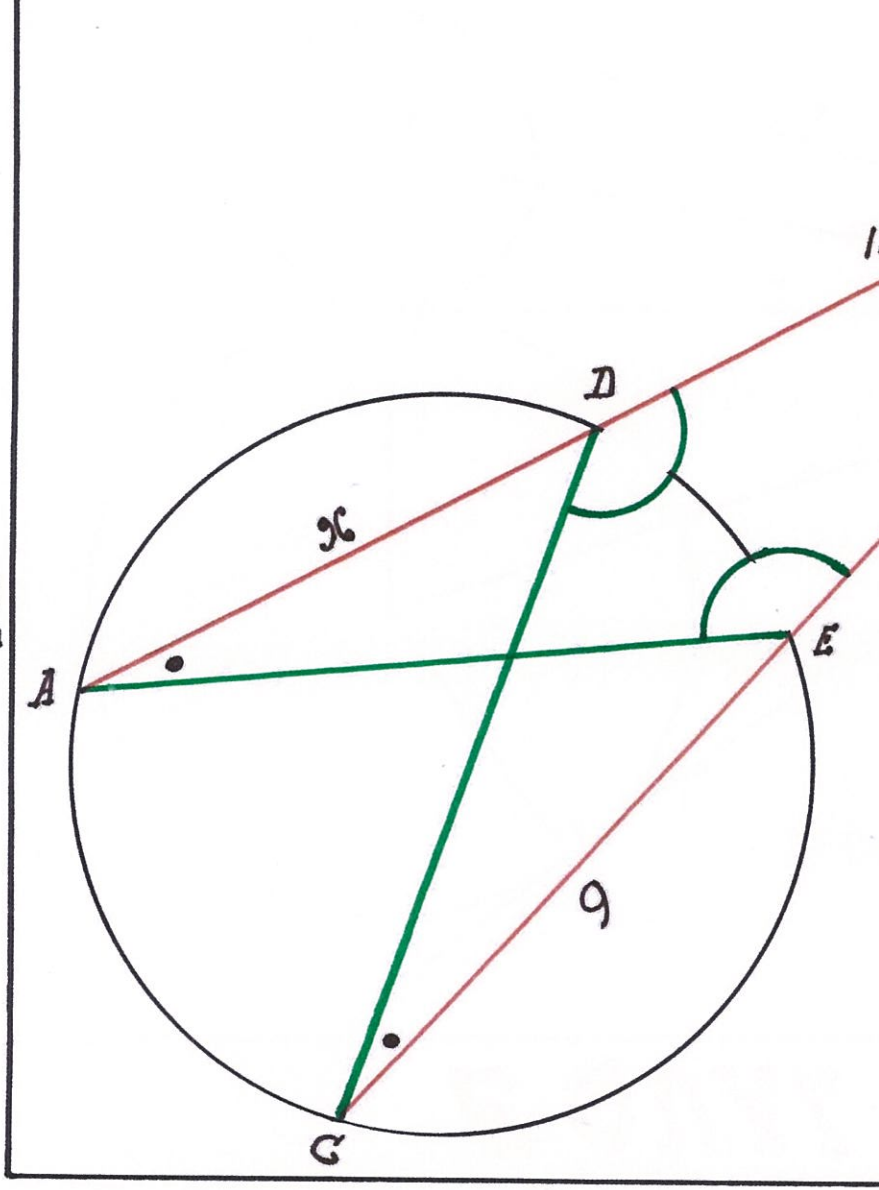
$AB = CD$



$\therefore$ (THEREFORE)

$EF = EG$

35



COMMON ANGLE

IF 2 ANGLES ARE EQUAL, THE 3RD ONE IS

$\hat{A} = \hat{C}$ (ON ARC DE). $\hat{B} = \hat{B}$

$\therefore$

$\triangle ABE \equiv \triangle CBD$ $\therefore$

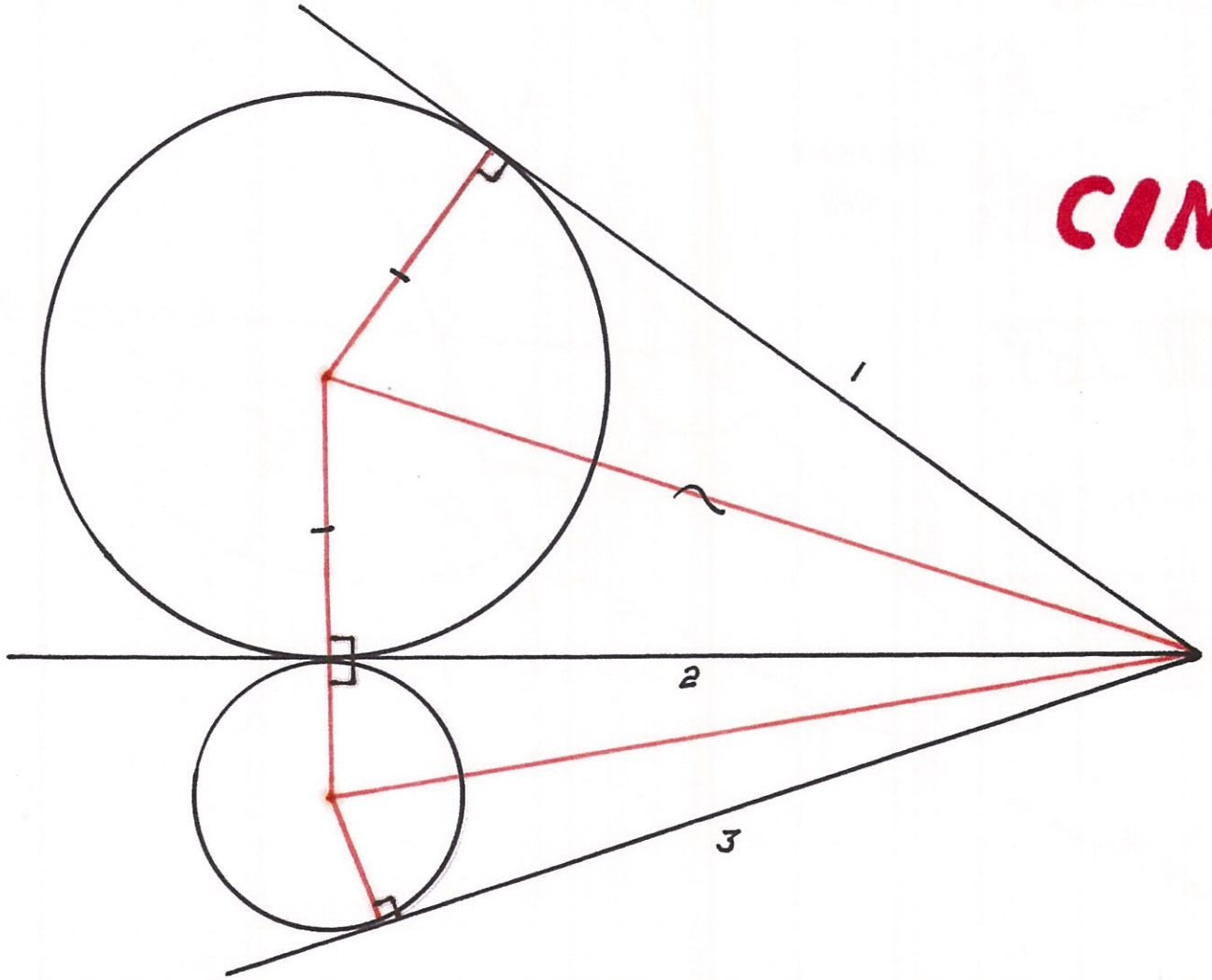
BOTH OPPOSITE

$$\frac{11}{11.5} = \frac{x+11.5}{20} = 19.1$$

BOTH OPPOSITE

$x = 7.6$

EQUAL TANGENTS

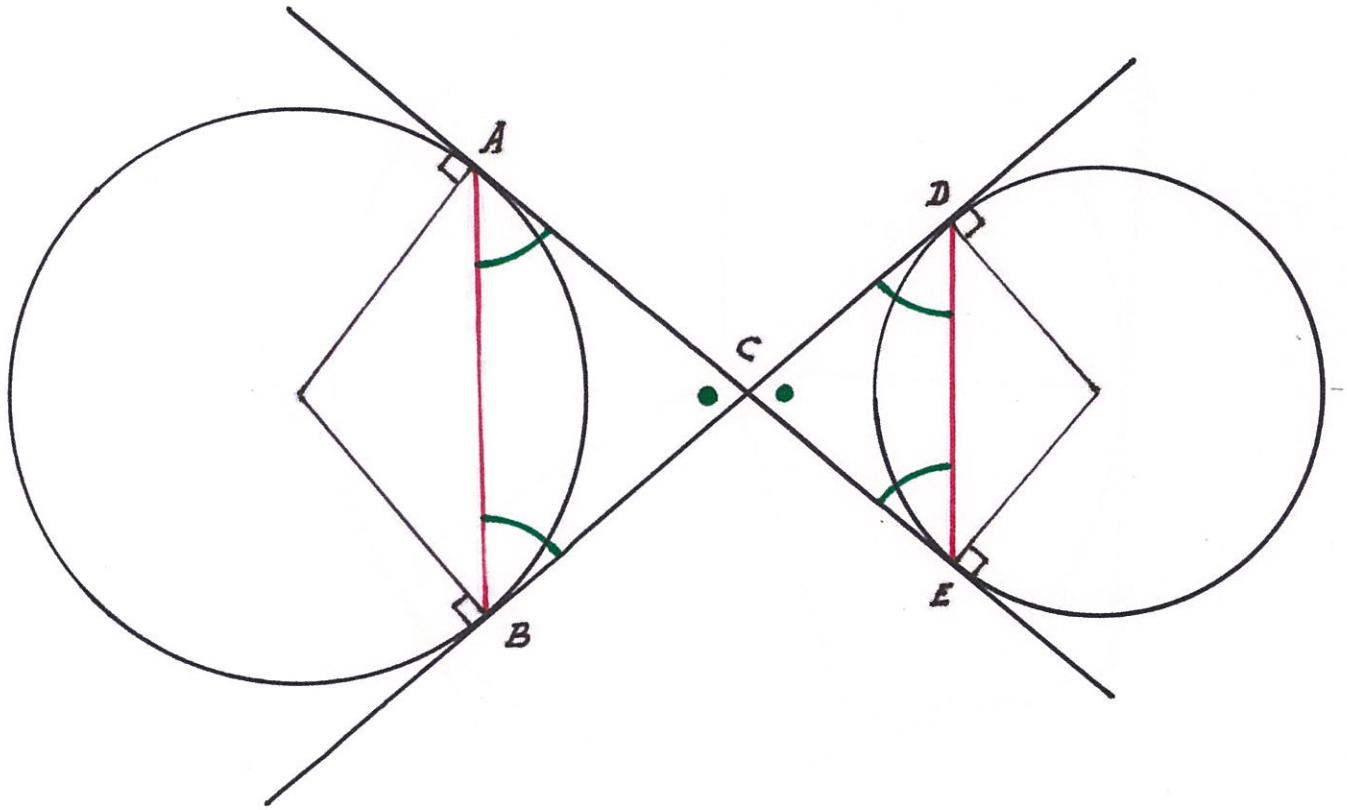


CONGRUENT TRIANGLES R.H.S

$\therefore$

$\left.\begin{matrix} 1=2 \\ 2=3 \end{matrix}\right\} \therefore 1=2=3$

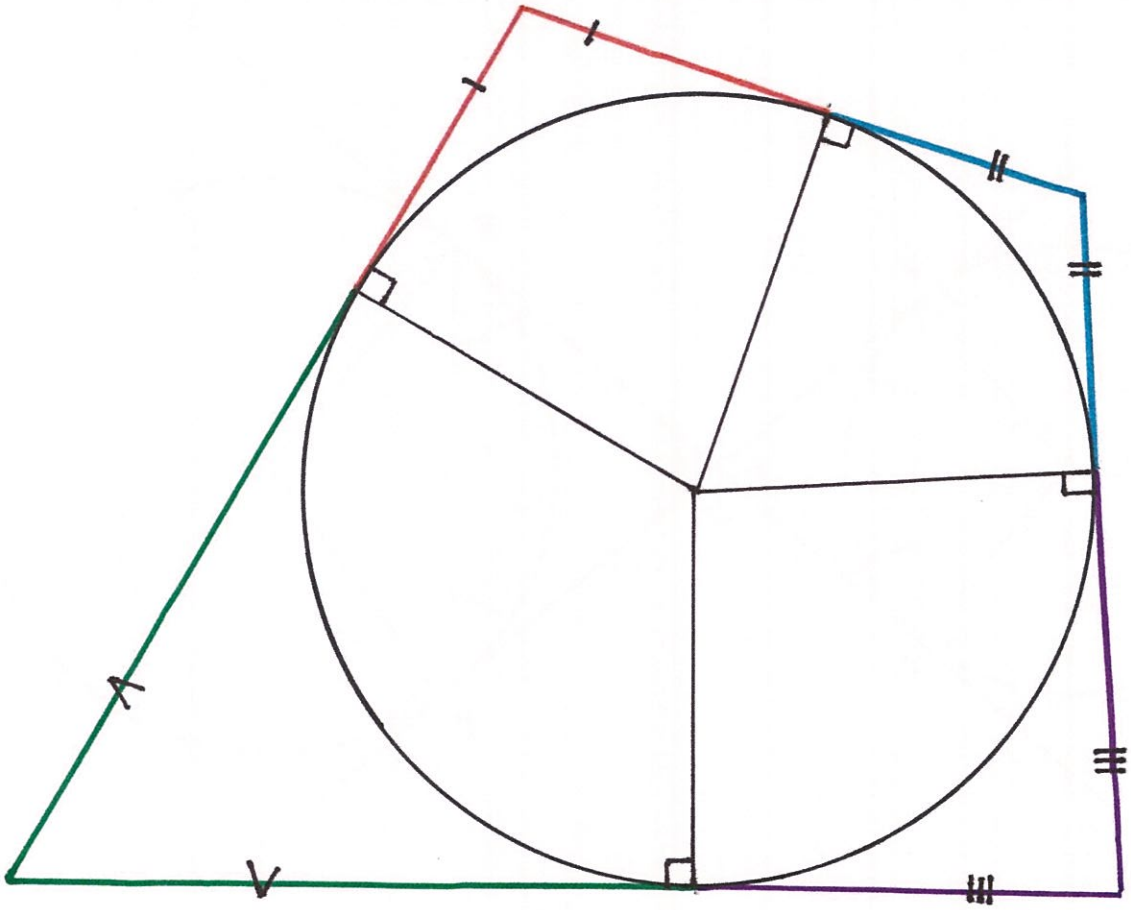
EQUAL TANGENTS, PARALLEL CHORDS



SIMILAR ISOSCELES TRIANGLES WITH EQUAL ALTERNATE ANGLES

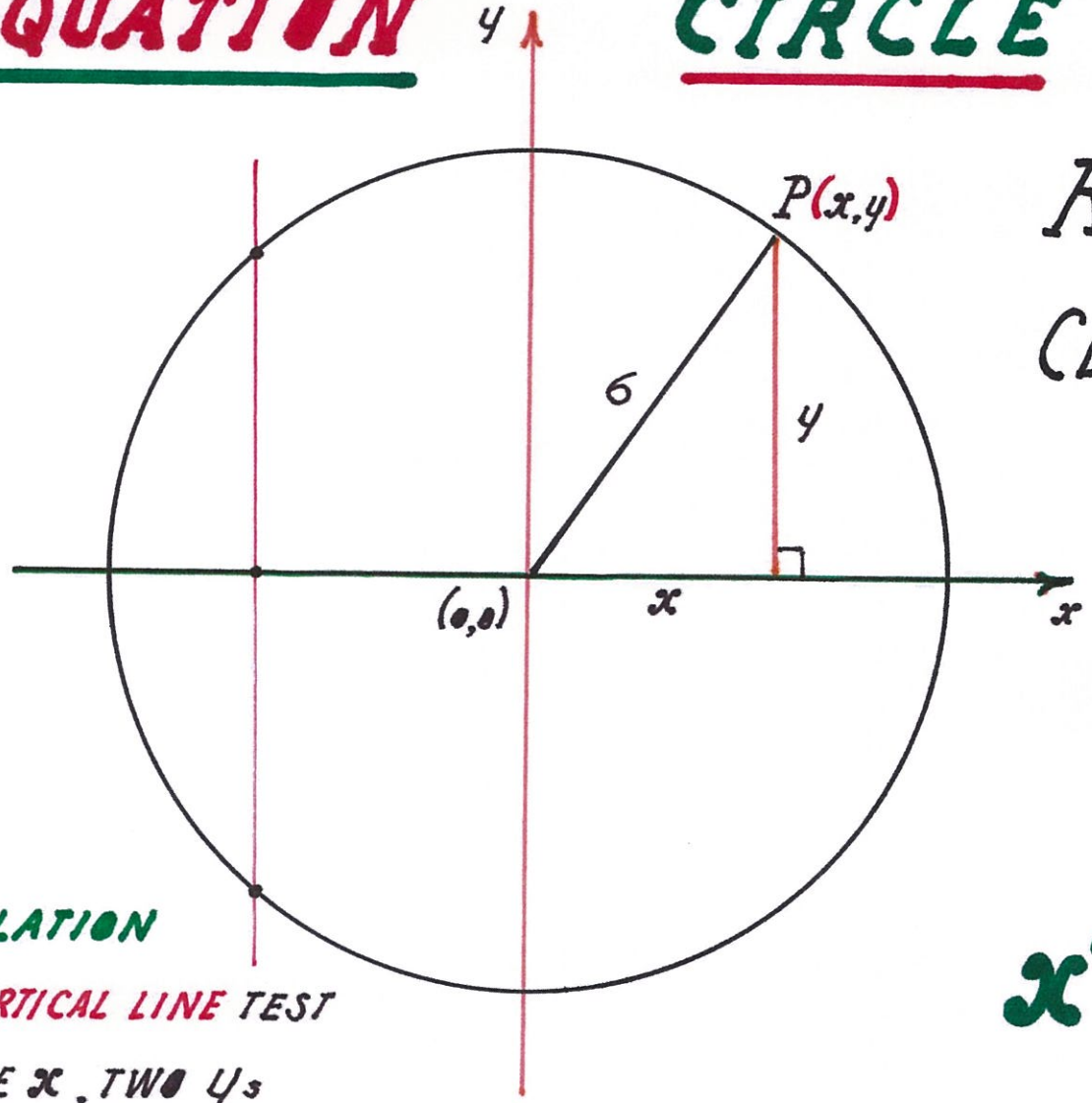
QUADRILATERAL CIRCUMSCRIBED ABOUT A CIRCLE

187.



SUM OPPOSITE SIDES EQUAL

EQUATION CIRCLE



RADIUS 6 cm

CENTRE (0,0)

THE ORIGIN

PYTHAGORAS

RELATION

VERTICAL LINE TEST

ONE x, TWO ys

$$x^2 + y^2 = 36$$

CIRCLES: TASK

31 ANGLES $2x, 3x, x, 6x$ | $\hat{A} + \hat{C} =$

32 IF $\overline{AD} + \overline{BC} = 20\,cm$, $\overline{AB} + \overline{DC} =$

33 $\overline{DE} = 3$, $\overline{EC} = 2$, $\overline{AB} = 4$, $\overline{AE} = 4$ | FIND x & y

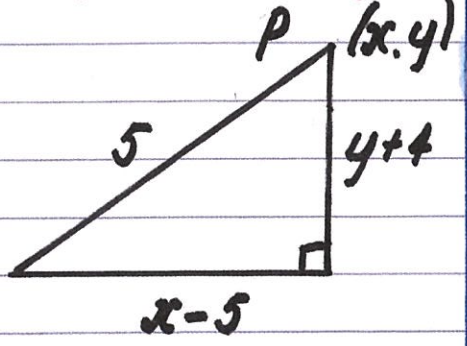
34 IF $\overline{AB} = 10$. $\overline{CD} =$ | IF $\overline{AF} = 4$ & $\overline{EF} = 3$, $\overline{AE} =$

35 $\overline{EB} = 6$. $\overline{DB} = 6$, $\overline{CE} = 5$, $x =$

39 EQUATION CIRCLE: $R = 8$, CENTRE ORIGIN $(0,0)$

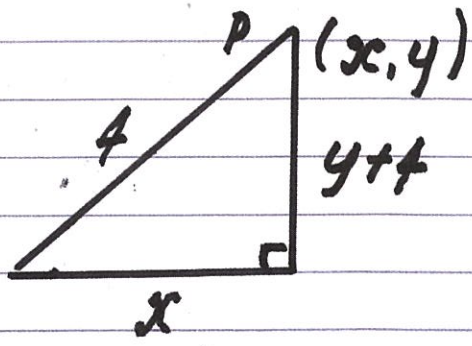
40 EQUATION CIRCLE: $R = 4$, CENTRE $(4,2)$

41 $R = 5$, CENTRE $(5,-4)$



EQUATION $(x-5)^2 + (y+4)^2 = 25$

$R = 4$, CENTRE $(0,-4)$



$x^2 + (y+4)^2 = 16$

EQUATION CIRCLE R_4, CENTRE $(4, -3)$

$$(x-4)^2 + (y+3)^2 = 16$$

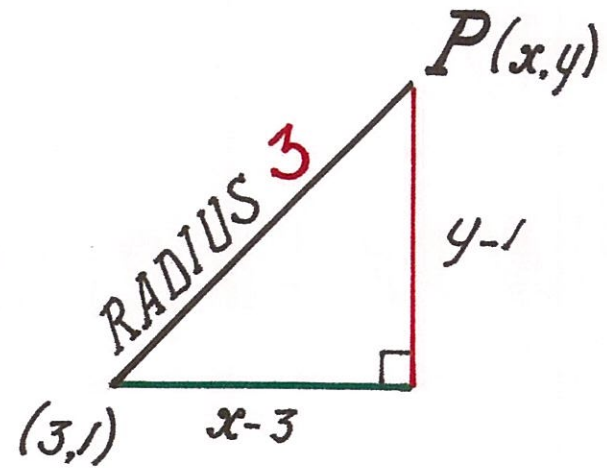
EQUATION CIRCLE R_5, CENTRE $(0, -5)$

$$x^2 + (y+5)^2 = 25$$

EQUATION CIRCLE

RADIUS 3 cm

CENTRE (3, 1)



SILLY TONGUE TWISTER

— PYTHAGORAS — ~~THEOREM~~

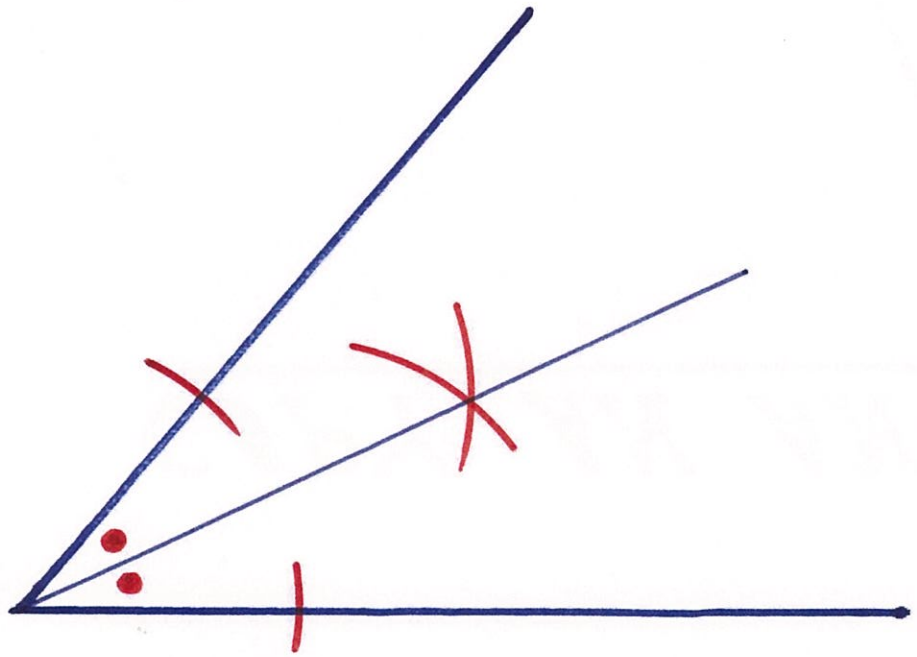
$$(x-3)^2+(y-1)^2=9$$

CONSTRUCTIONS

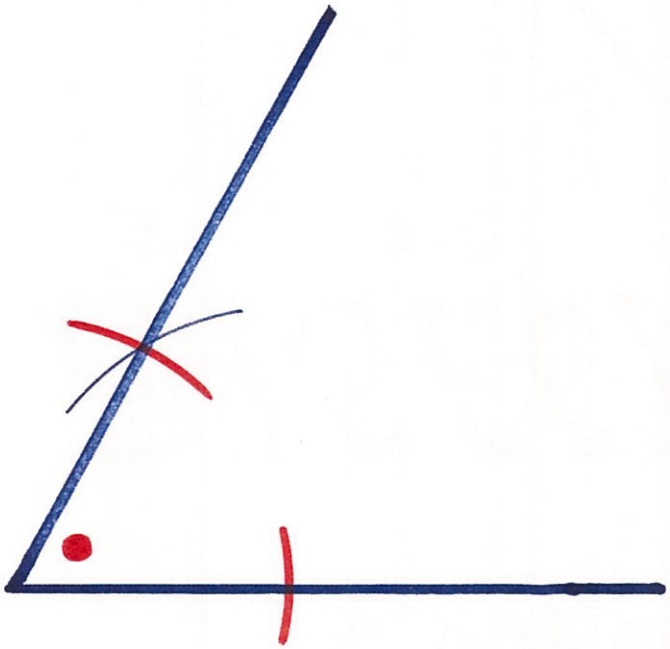
BISECT AN ANGLE

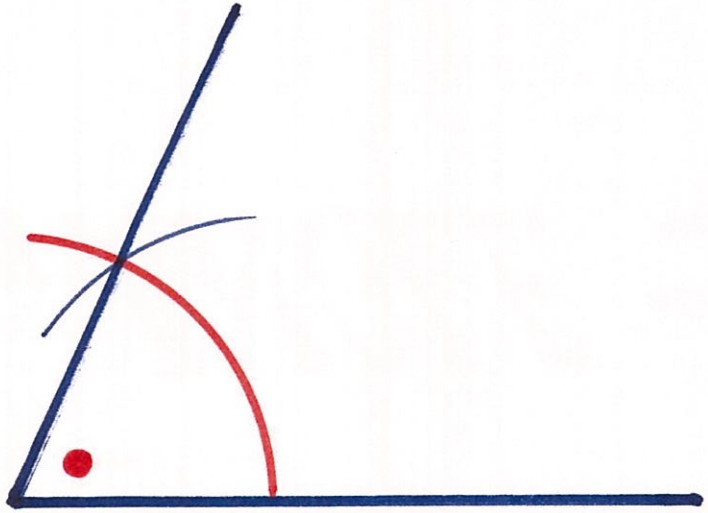
NOT AN ANGEL!

2 CUT

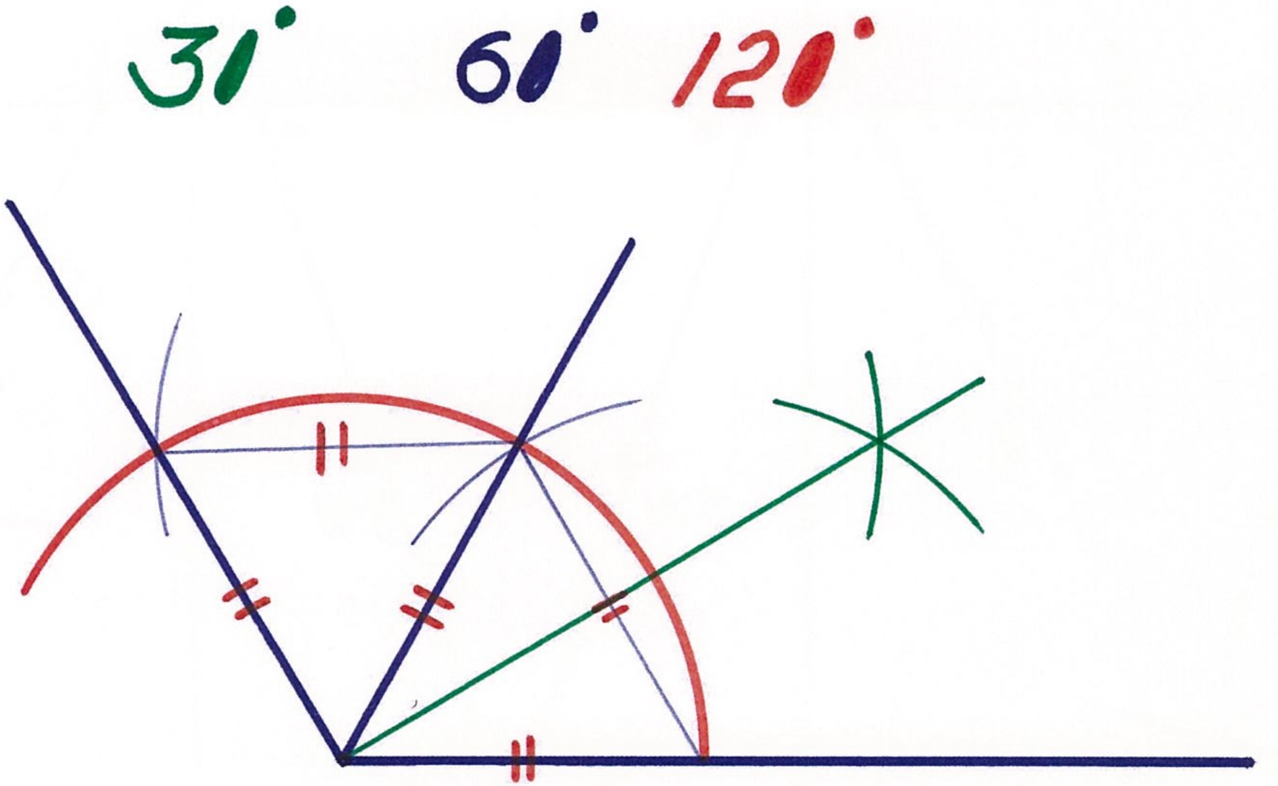


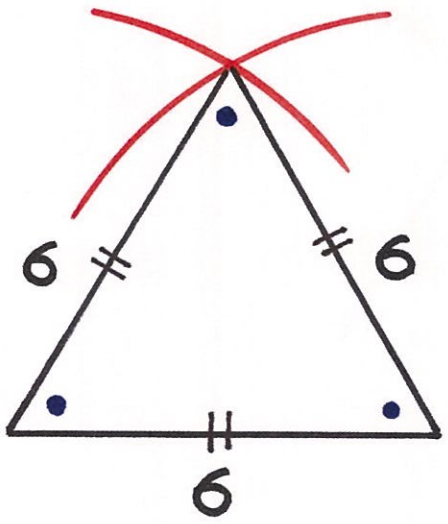
COPY AN ANGLE





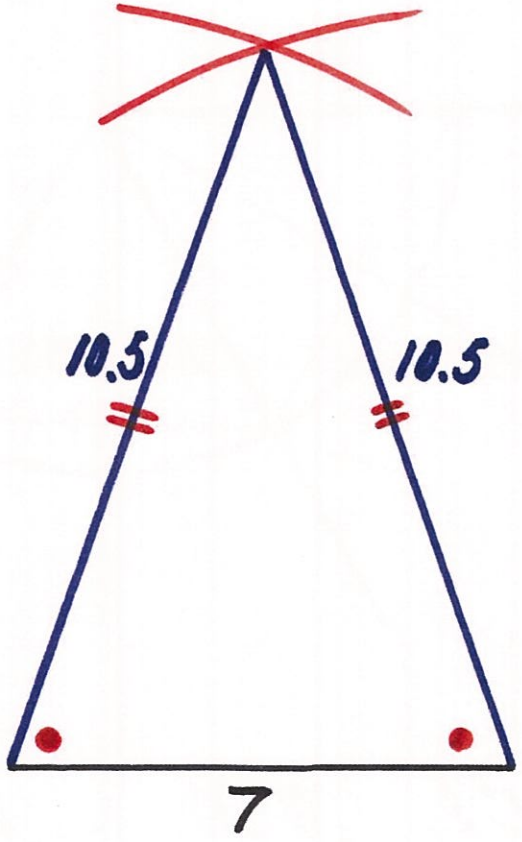
225.





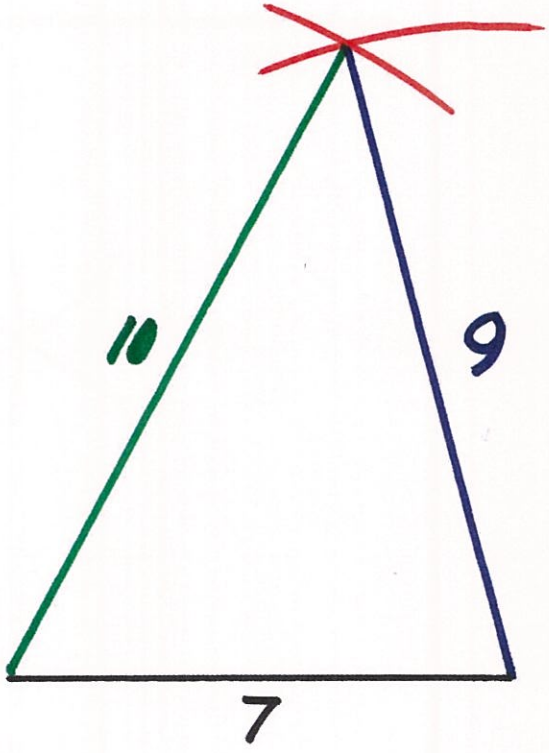
EQUILATERAL

EQUAL SIDES (LATIN)



ISOSCELES

EQUAL LEGS (GREEK)



SCALENE

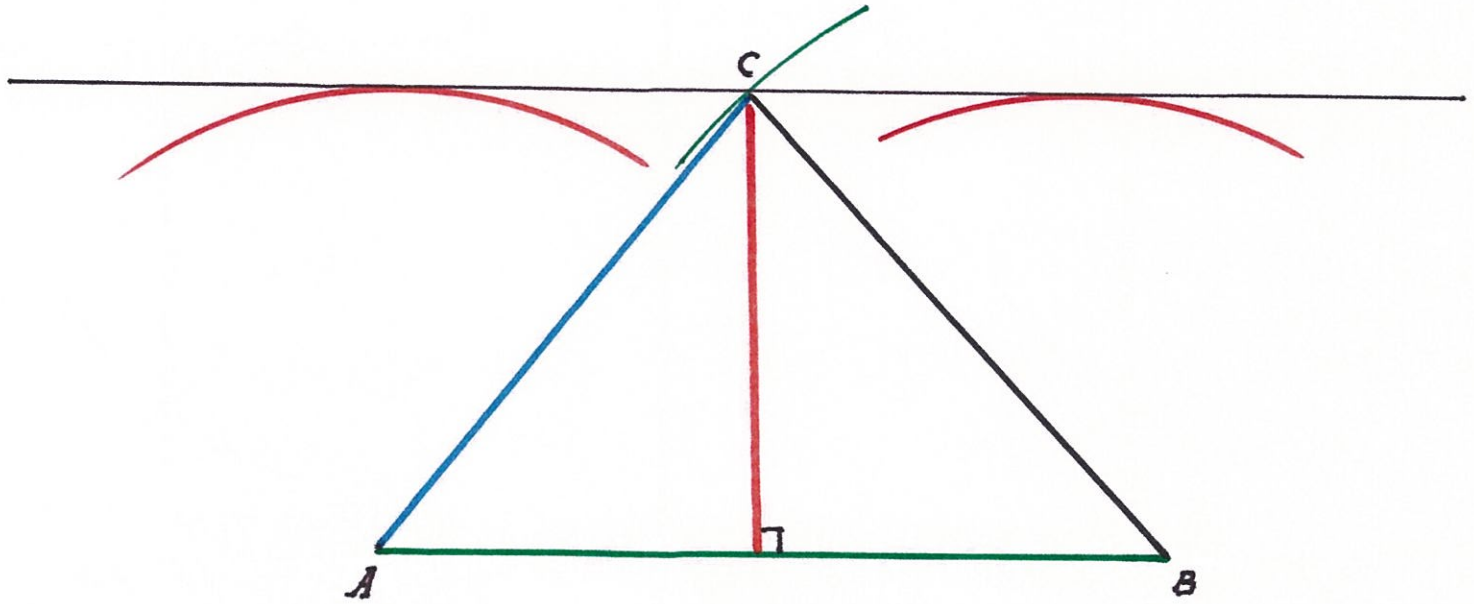
227.

GIVEN

SIDE

BASE

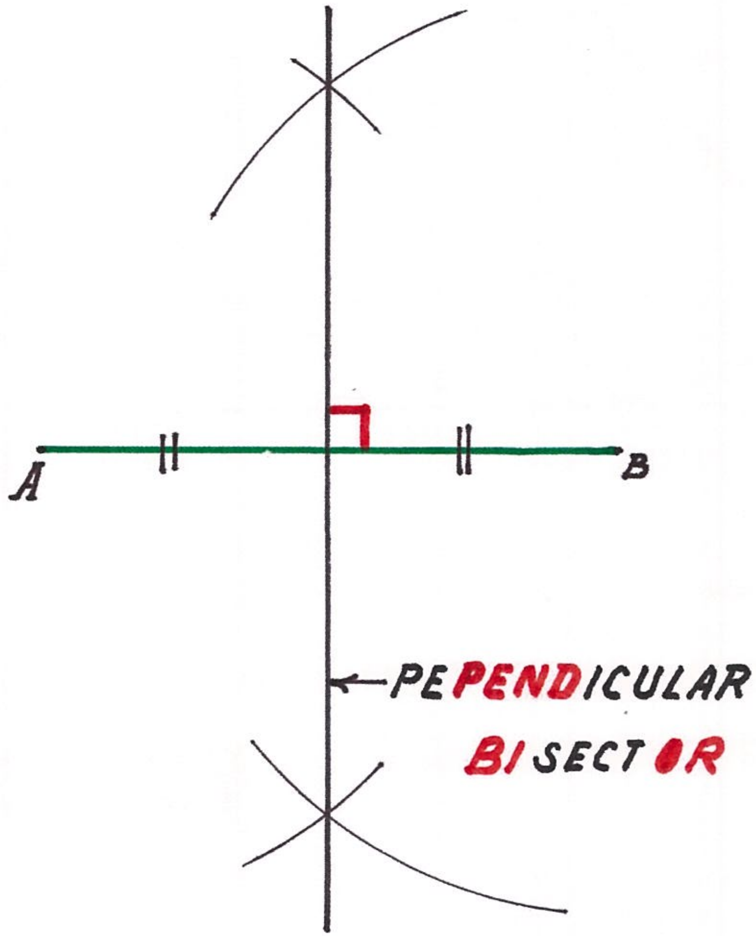
ALTITUDE (HEIGHT)



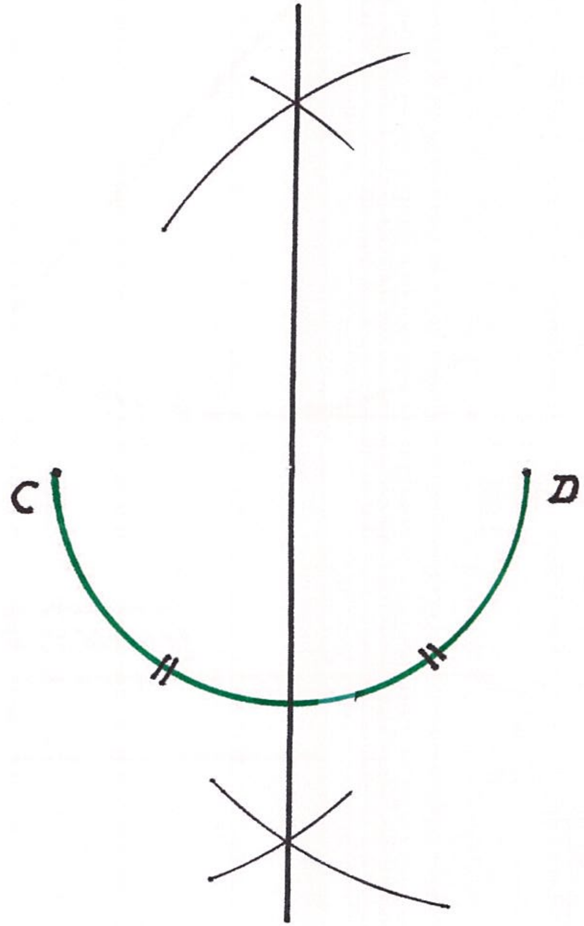
228.

BISECT AN INTERVAL

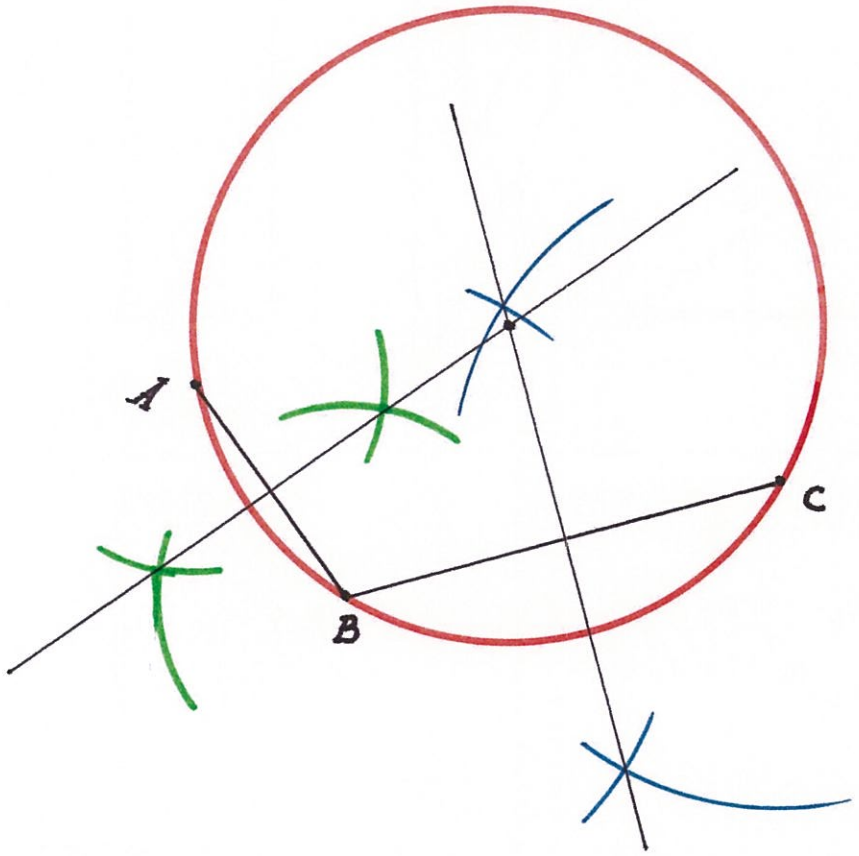
LINE SEGMENT



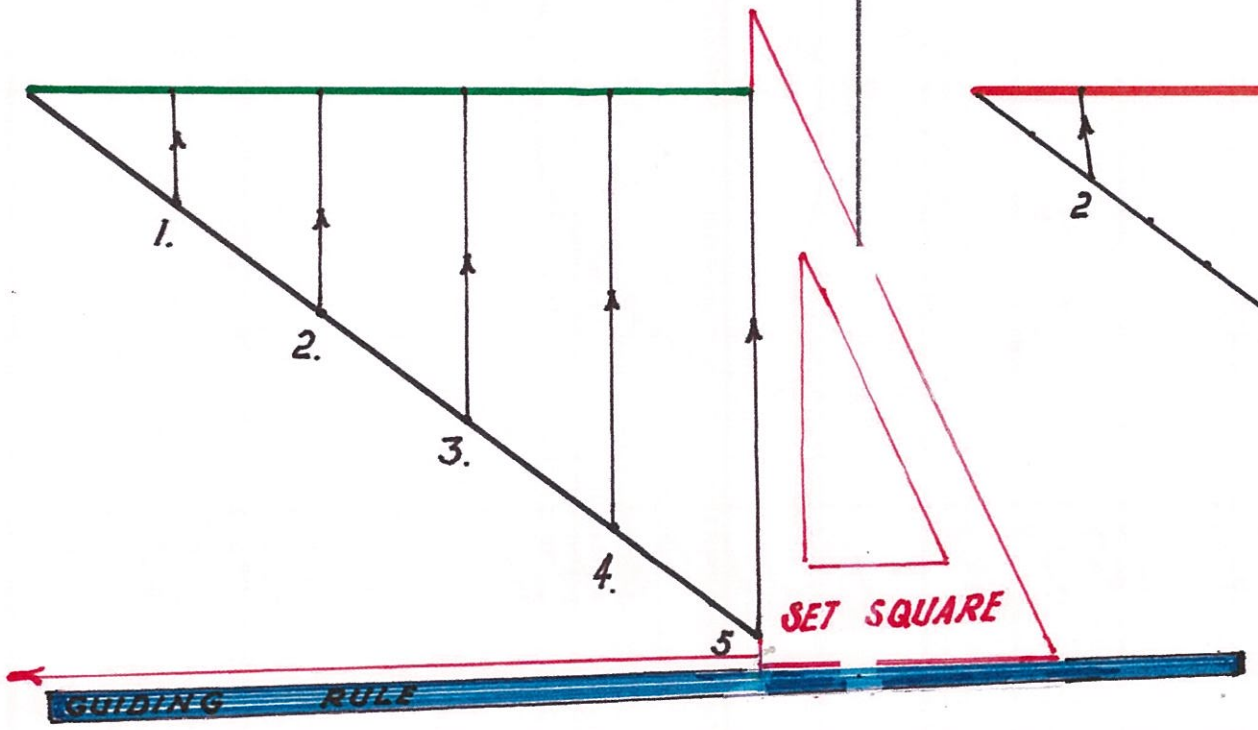
BISECT AN ARC



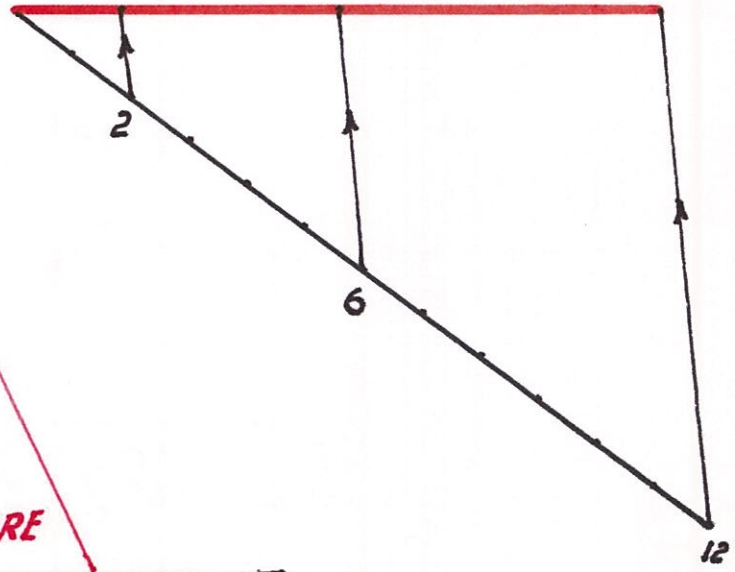
A CIRCLE THROUGH 3 POINTS



DIVIDE AN INTERVAL INTO 5 EQUAL PARTS



DIVIDE AN INTERVAL INTO 3 PARTS RATIO 1:2:3

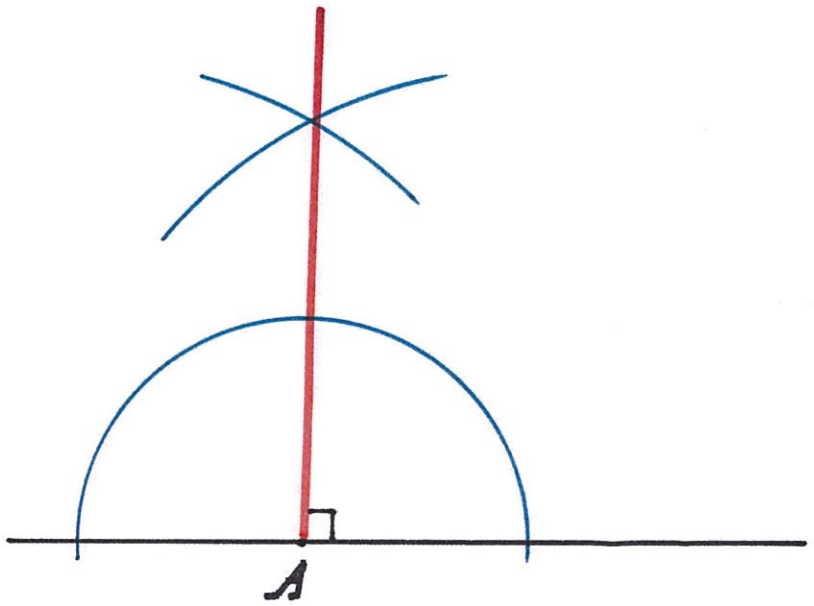


50

231.

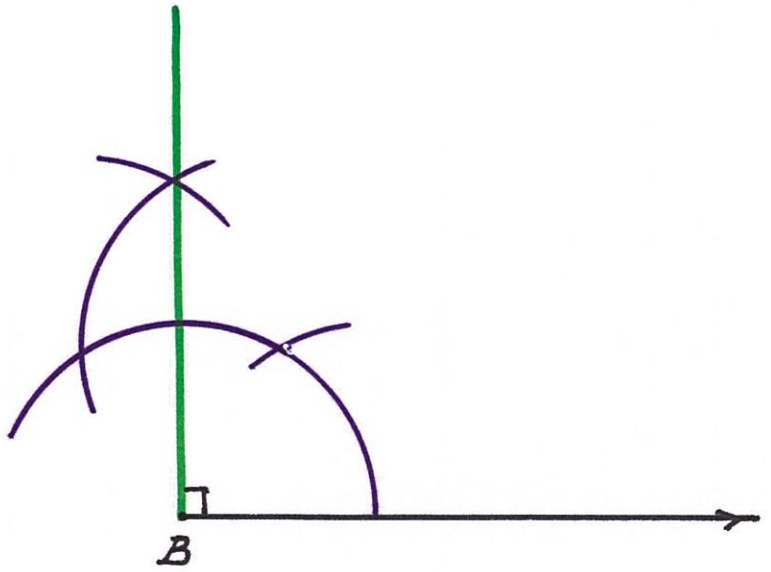
PERPENDICULAR

FROM A POINT IN A LINE



PERPENDICULAR

FROM THE BEGINNING OF A RAY.



	CONSTRUCTIONS: TASK	1.
223.	BISECT AN ANGLE	
224.	COPY AN ANGLE	
225.	ANGLES: 30°, 60°, 120°	
226.	EQUILATERAL, ISOSCELES, SCALENE	
227.	TRIANGLE: GIVEN: SIDE, BASE, ALTITUDE.	
228.	BISECT AN INTERVAL & AN ARC.	
229.	A CIRCLE THROUGH 3 POINTS	
230.	DIVIDE AN INTERVAL INTO 5 EQUAL PARTS DIVIDE AN INTERVAL INTO 3 PARTS RATIO 1:2:3	
231.	PERPENDICULARS FROM POINTS IN A LINE & FROM THE BEGINNING OF A RAY.	

271.

$$\frac{1}{x} + 3 = \frac{4}{x}$$

THINK

$1 + 3x = 4$

$x = 1$

$$\frac{2x}{5} + x = \frac{x+4}{2}$$

$\times 10$

$4x + 10x = 5x + 20$

$x = 2\frac{2}{9}$

271.

$$\frac{5x-1}{4} = 1 + \frac{x+1}{5}$$

$\times 20$

$25x - 5 = 20 + 4x + 4$

$x = 1\frac{8}{21}$

$$\frac{x+\frac{1}{2}}{4} = 4x + \frac{3x}{2} - \frac{5}{2}$$

$x + \frac{1}{2} = 16x + 6x - 10$

$x = \frac{1}{2}$

272.

$$\frac{5}{8}(x+4) = 4x - \frac{1}{2}$$

$5x+20 = 32x-4$

$x = \frac{8}{9}$

$$\frac{x}{4} - \frac{9}{2} = 1$$

$x-18 = 4$

$x = 22$

273.

$$\left(\frac{x+1}{3} - \frac{x}{7} = \frac{x+4}{5}\right) \times 105$$

$$\overset{5\times7}{35}(x+1) - \overset{3\times5}{15}x = \overset{3\times7}{21}(x+4)$$

$$35x + 35 - 15x = 21x + 84$$

$$x = -49$$

274.

$$\frac{x+6}{2} - \frac{x-2}{5} = \frac{x+9}{3}$$

$^{5\times3}15(x+6) - {}^{2\times3}6(x-2) = {}^{2\times5}10(x+9)$

$15x+90 - 6x+12 = 10x+90$

$x = 12$

NOTICE **BEFORE** YOU START!

$$\frac{3}{4}(x+3)-\frac{1}{3}(2x-3)=\frac{5}{6}$$

USE L.C.M **12**

$9(x+3)-4(2x-3)=11$

$9x+27-8x+12=11$

$x=-29$

QUADRATIC EQUATIONS: 2 ANSWERS

$15\uparrow x^2 + 16x - 15 = 0$

FACTORISED: $(5x - 3)(3x + 5) = 0$

OPPOSITE & DIVIDE BY 5 → $x = \frac{3}{5}$

OPPOSITE & DIVIDE BY 3 → $x = -1\frac{2}{3}$

59

$8x^2 + 12x - 8 = 0$

FACTORISED: $(2x + 4)(4x - 2) = 0$

$x = -2$ $\quad$ $x = \frac{1}{2}$

TRIANGLE: $P = 40$ cm

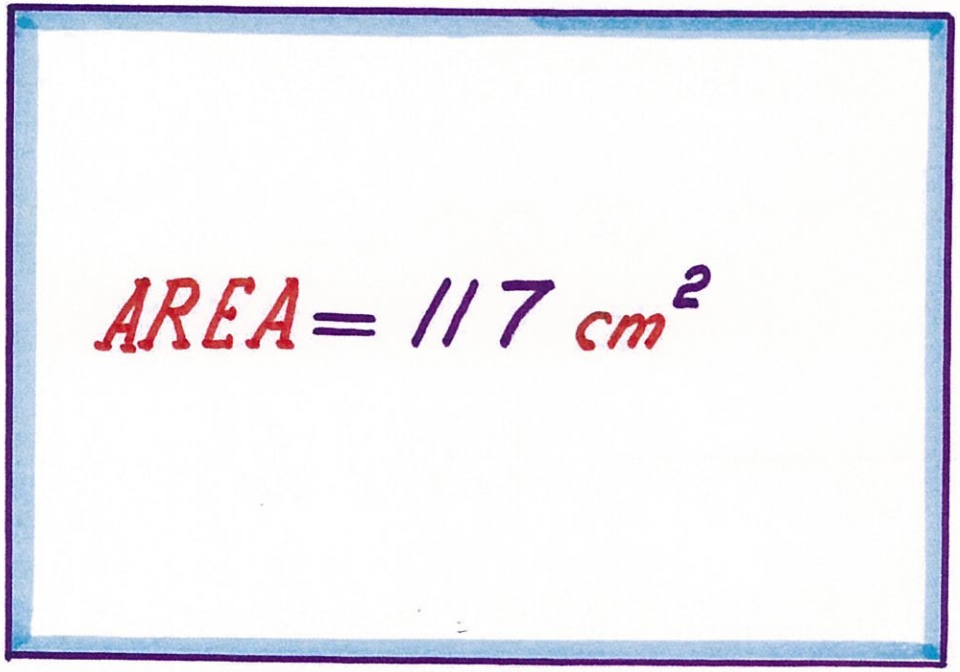
SIDES $(4x-3)$, $(5x+2)$, $(3x+5)$

$$12x + 4 = 40$$

$x = 3$ ∴ SIDES ARE 9, 17 & 14 cm

RECTANGLE

HEIGHT 9 cm, $P = 44$ cm



$\therefore$

$2B = 26$

$B = 13$ cm

INDIRECT VARIATION

3 CATS ⟶ FOOD FOR 12 DAYS

1 CAT ⟶ 36 DAYS. 4 CATS ⟶ 9 DAYS (NOT 16!)

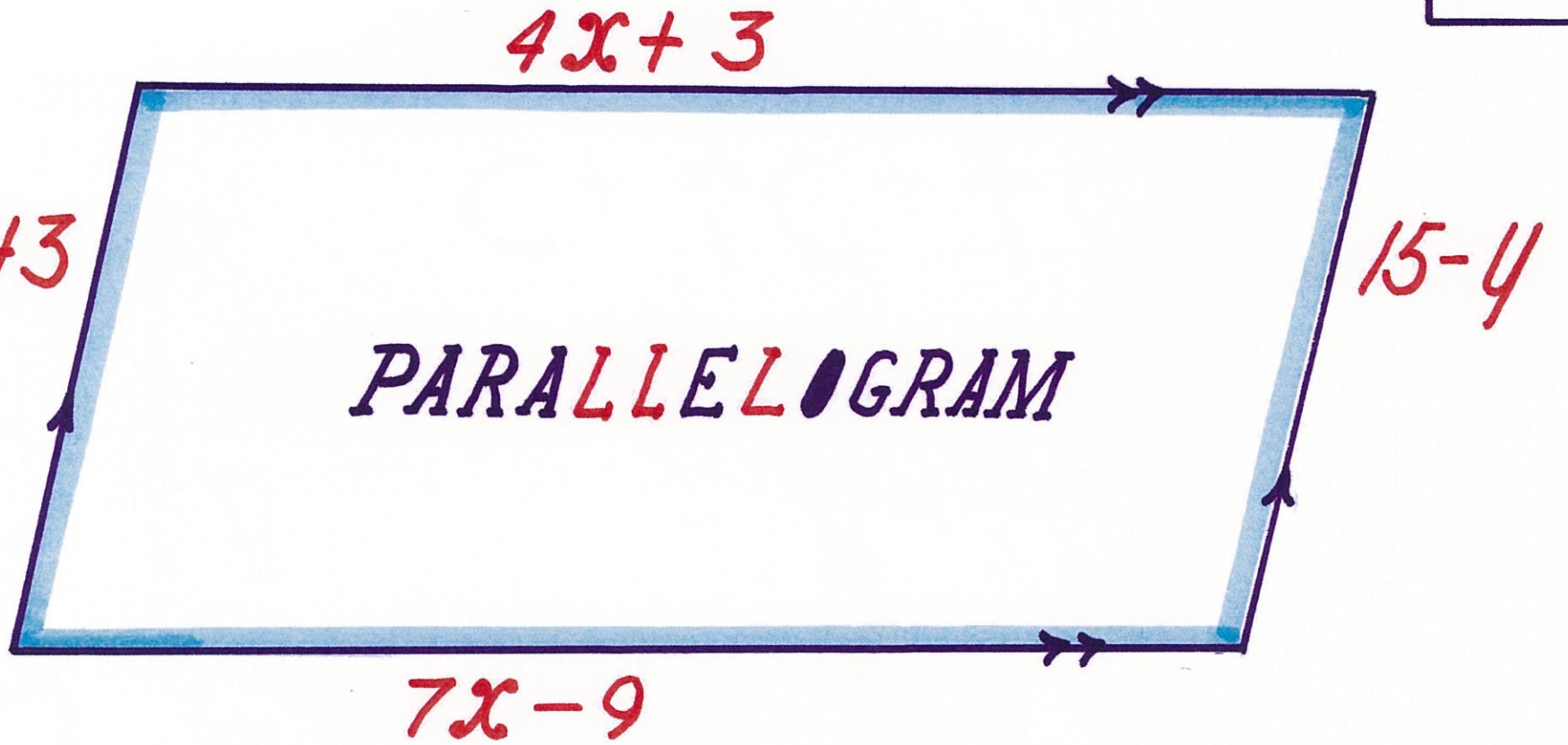
$\frac{1}{2}$ YEAR 10 PLAYS TENNIS, $\frac{1}{3}$ CRICKET.

40 STUDENTS NO SPORT

SPORT: 

$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ ∴ $\frac{1}{6}x = 40$ ∴ 240 STUDENTS IN YR 10

280.



$7x-9=4x+3$

$x=4$

$y+3=15-y$

$y=6$

3 PEOPLE, 118 BOOKS

$A = \frac{1}{2}B$ $\qquad$ $B = \frac{1}{3}C$

$$C + \frac{1}{3}C + \frac{1}{6}C = 118$$

$\frac{3}{2}C = 118,\ C = 72$

$\therefore B = 24,\ A = 12$

RECTANGLE $(3x+2)$ BY $(2x-4)$. $P = 76$ cm

$10x - 4 = 76 \longrightarrow x = 8 \longrightarrow$ AREA $= 312$ cm^2

TANK $\frac{5}{8}$ FULL. 270 LITRES TO FILL

$$\frac{3}{8} \equiv \frac{270}{x} \quad (\times 8x) \quad x = 720 \text{ LITRES}$$

CAPACITY TANK

JO IS x YEARS	3 YEARS AGO, $x-3$	IN 4 YEARS, $x+4$	283.

10 YEARS AGO, A WAS 3 TIMES B'S AGE; NOW 2 × B'S AGE

$(A-10)=3(B-10)$ $\therefore$ $A=3B-20$ (1) $=2B$ (2) $\therefore B=20$ YRS

(SIMULTANEOUS EQUATIONS)

$A=40$ YRS

99

ANGLE RATIO TRIANGLE $1:3:5$

$9x=180^\circ$ $\therefore x=20^\circ$

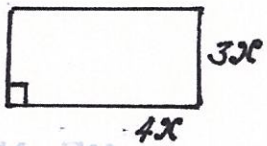
ANGLES ARE 20°, 60° & 100°

284.

RECTANGLE $P = 160$ cm $B = 3H$

$8H = 160$, $H = 20$ cm, $B = 60$ cm , AREA $= 1200$ cm^2

RECTANGLE $P = 238$ cm $B : H = 4 : 3$



$14x = 238$, $x = 17$, AREA $= 68 \times 51 = 3468$ cm^2

EQUATIONS: TASK 1.

53

$$\frac{1}{2x} + 4 = \frac{3}{2x}$$

$$\frac{x}{5} + \frac{x}{2} = \frac{2x+3}{4}$$

54

$$\frac{4x-2}{5} = 1 + \frac{x-1}{3}$$

$$\frac{x+\frac{1}{3}}{5} = 2x + \frac{2x}{3} - \frac{4}{3}$$

55

$$\frac{5}{6}(2x+1) = 2x - \frac{1}{3}$$

$$\frac{x}{9} - \frac{1}{3} = 2$$

56

$$\frac{2x+1}{3} - \frac{x}{4} = \frac{x+3}{7}$$

58

$$\frac{3}{4}(2x-1) - \frac{1}{3}(2x-4) = \frac{5}{8}$$

EQUATIONS: TASK 2.

59 $(4x-5)(5x+7)=0$

60 TRIANGLE: SIDES $(3x-4), (2x+5), (5x+3)$ $P = 44\ cm$

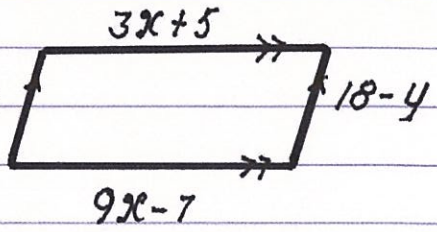
61 RECTANGLE: $H = 10\ cm$, $P = 60\ cm$

FIND B & A

62 6 CATS: FOOD FOR 18 DAYS: 9 CATS?

$\frac{1}{3}x \rightarrow$ TENNIS, $\frac{1}{4}$ CRICKET. No SPORT 10

63 PARALLELOGRAM



64 3 PEOPLE, \$16

$A = \frac{1}{3}B$, $B = \frac{1}{4}C$

EQUATIONS: TASK 3

65 RECTANGLE $(2x+3)$ BY $(4x-2)$. $P = 38$ cm

66 JO IS x YEARS

5 YEARS AGO, HE WAS ...

IN 10 YEARS, HE WILL BE ...

8 YEARS AGO. A WAS $4x$ B

NOW 3B

ANGLE RATIO TRIANGLE

$1:2:3$

67

RECTANGLE: $P = 180$ cm

B $= 2H$

RECTANGLE: $P = 100$ cm

B $: H = 3:2$

THE DIFFERENCE OF 2 CUBES

$$a^3 - b^3$$

$$(a - b)(a^2 + ab + b^2)$$

SINGLE PRODUCT

$$64a^3 - 8 = (4a - 2)(16a^2 + 8a + 4)$$

THE SUM OF 2 CUBES

$$a^3 + b^3$$

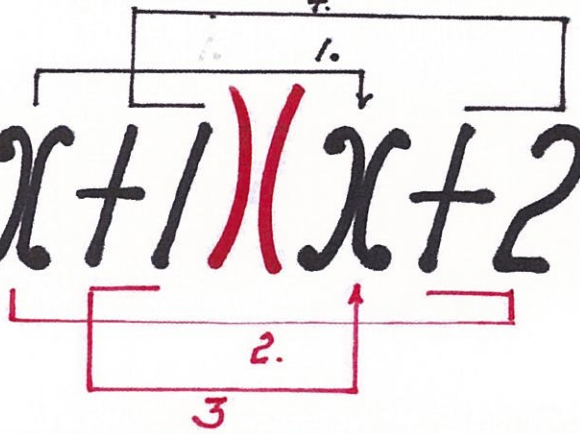
$$(a + b)(a^2 - ab + b^2)$$

EXPANDING = MULTIPLICATION

321.

4 STEPS : COMBINE STEPS 2 & 3 MENTALLY

MEANS SEE BEFOREHAND !



$$(x+1)(x+2) = x^2+3x+2$$

$$(x-3)(x+5) = x^2+2x-15$$

321.

$$(x-4)(x-6)=x^2-11x+24$$

$$(6x-1)(x-3)=6x^2-19x+3$$

$$(3x-1)(2x+4)=6x^2+11x-4$$

FACTORISING TRINOMIALS

322.

$$x^2 - x - 12$$

END PRODUCT

END PRODUCT NEGATIVE ∴
ONE FACTOR IS POSITIVE
ANOTHER IS NEGATIVE
THE NEGATIVES WIN BY 1

STEP 1. $(x \quad)(x \quad)$

STEP 2. $(x - \quad)(x + \quad)$

STEP 3. $(x - 4)(x + 3)$

TYPE 1.

$A = 1$

$$x^2 - 15x + 56$$

$$Ax^2 + Bx + C$$ GENERAL FORM

2 POSITIVE FACTORS NOT APPLICABLE

2 NEGATIVE FACTORS

NEGATIVES ARE WINNING

STEP 1. $(x \quad)(x \quad)$

STEP 2. $(x - \quad)(x - \quad)$

STEP 3. $(x - 7)(x - 8)$

324.

$$x^2+5x+6=(x+2)(x+3)$$

77

$$x^2-2x-8=(x-4)(x+2)$$

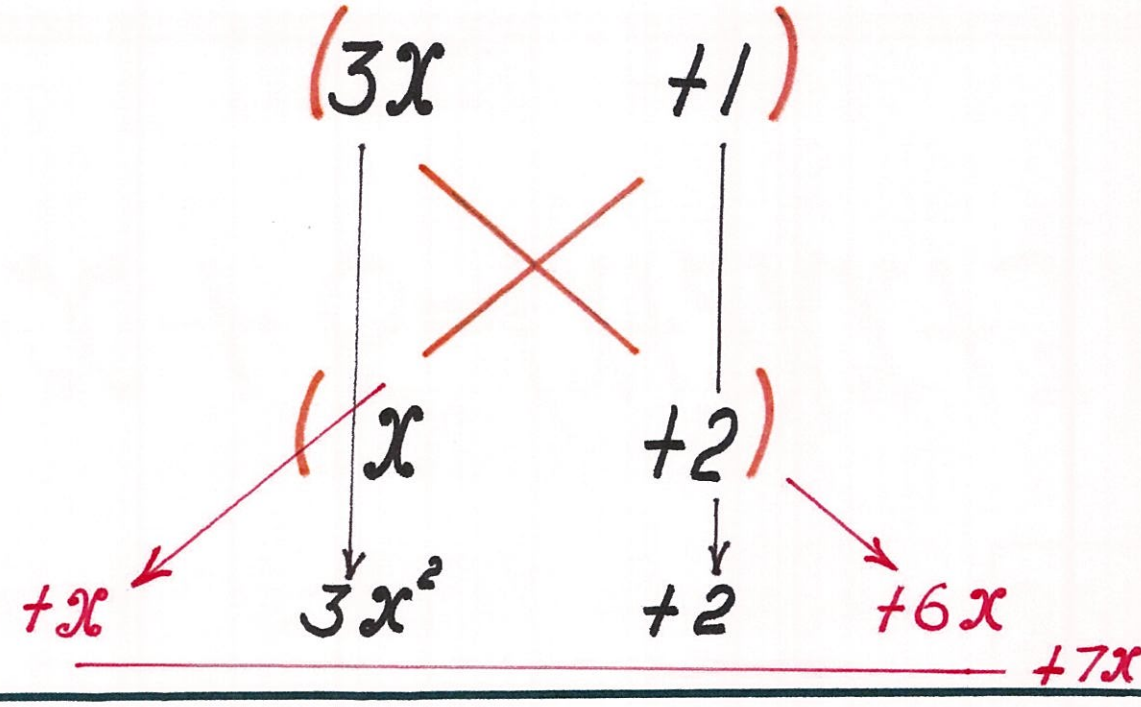
TYPE 2: A IS PRIME

$$3x^2+7x+2 = (3x+1)(x+2)$$

$+x+6x$

THE CROSS METHOD

TRIALS ERROR



64

326.

$2x^2 - 5x - 12 =$

$-8x + 3x$

$-8x$ ↖ $(2x + 3)$ ↗ $+3x$

$(x - 4)$

$3x^2 - 7x + 2 =$

$(3x - 1)$

$(x - 2)$

TYPE 3: A COMPOSITE

327.

$$6x^2+11x-4=(3x-1)(2x+4)$$

$$4x^2-2x-6=(2x+2)(2x-3)$$

EXAMPLE: $(89 \times 13) + (11 \times 13) = 100 \times 13$ | 328.

$$x(x-2) + 3(x-2) = (x+3)(x-2)$$

NOT WRITTEN ↓

$$(x+y) - z(x+y) = (1-z)(x+y)$$

4 TERMS: AN EXTRA STEP

$$ab^2 + ac - b^2d - cd$$

$$a(b^2+c) - d(b^2+c) = (a-d)(b^2+c)$$

BINOMIAL

$$\frac{x^2-5x}{x^2-25}$$

THE DIFFERENCE OF 2 SQUARES

ONE SET

$$\frac{x(x-5)}{(x-5)(x+5)} = \frac{x}{(x+5)} \qquad x \neq 5$$

331.

$$\frac{x(x-2)+3(x-2)}{x^2-4}$$

$$\frac{(x-2)(x+3)}{(x-2)(x+2)} = \frac{(x+3)}{(x+2)}$$

$x \neq 2$

EXPRESSIONS: TASK 1.

71 $125a^3 - 27$

72 $125a^3 + 27$

73 $(x+2)(x+3)$

$(x-4)(x+3)$

74 $(x-5)(x-7)$

$(2x-2)(3x+5)$

75 $x^2 - 8x + 15$

76 $x^2 - 2x - 24$

78 $15x^2 + 9x - 6$

80 $3x^2 - 7x + 2$

79 $2x^2 - 5x - 12$

INSTEAD OF TRIAL & ERROR

USE $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ SUB $a=2, b=-5$ $c=-12$

$\frac{5 \pm \sqrt{25+96}}{4}$

$x = 4$ $\quad x = \frac{3}{2} \therefore 2x = 3$ $\quad 2x - 3 = 0$

$\therefore 2x^2 - 5x - 12 = (x-4)(2x+3)$

81 $x(x-3) + 3(x-\ )$

$a^2b + bc - bd + ac$

82 $\frac{x^2 - 3x}{x^2 - 9}$

83 $\frac{x(x-4) + 2(x-4)}{x^2 - 16}$

INTEREST ON HOMELOAN (ANNO 1910)

FROM 7.5% TO 8.27% P.A.

INCREASE $71.25 P.A.

IF .77% OF THE LOAN = 71.25

THE LOAN = 71.25 ÷ .0077 = $9253

$15990 CASH OR

30% DEPOSIT + 18 X $680
MONTHS

HOW MUCH MORE & AT WHAT %

DEPOSIT + PAYMENTS − CASH AMOUNT
3 X 1599 + 18 X 680 − 15990 = $1047 MORE

LOAN IS ON 7 X 1599 = $11193

WHAT % IS THIS OF THAT
1047 ÷ 18 X 12 ÷ 111.93 = 6.24%

86

WAGES: $11 PER HOUR

38 HOURS NORMAL, 4 HOURS DOUBLE TIME,

3 HOURS TIME-AND-A-HALF (= $1\frac{1}{2}$ x)

ADD THE HOURS FIRST: 51.5

AMOUNT BEFORE TAX $515

THE 9-PARTNERS

1	8
2	7
3	6
4	5

ONE YEAR

365 DAYS

ONE YEAR

52 WEEKS

7 DAYS, 1 WEEK

26 FORTNIGHTS

HOW TO REMEMBER

CHOOSE ONE ONLY

1. 5 FINGERS 2 HANDS
2. 5 SCHOOLDAYS 2 DAYS OFF

AWARD

OVERTIME RATES

356.

$12 PER HOUR FIRST 8, 1 1/2 NEXT 2, DOUBLE THEREAFTER

9 HOURS $114 96+18	11 HOURS $156 96+36+24

68

HOLY DAY

HOLIDAY LOADING

$16952 P.A. 4 WEEKS + 17.5%

100%

16952 ÷ 13 X 1.175 = $1532.20

AS A DECIMAL

CASUAL RATES

NO HOLIDAY PAY, NO SICK LEAVE

$12 + 20% PER HOUR ($14.40)

100% + 20% = 120%
= 1.2

6 HOURS: 6X14.4 OR 7.2X12 = $86.40

NORMAL RATE $12 P.H.

WEEKEND RATES: 6 HOURS

SAT. 2 AT 1 1/2, THEN DOUBLE	SUN. DOUBLE TIME
$132	$144

06

PIECE WORK

1550 @ 10 CENTS EACH: $155

16

INDEXATION

3.2% INCREASE ON $500 P.W.

DIFFERENCE PER ANNUM: 52 X 16 = $ 832

359.

COMMISSION

$125 RETAINER P.W. + 8.5% ON $2000 SALE

EARNINGS $295 (125+2X85)

26

SUCCESSIVE DISCOUNTS

TRADE 15% + 3% IF PAID WITHIN 14 DAYS

PAINT $366.20 PAY 366.2X.85X.97=$301.93

CONVERSION

REDUCIBLE (INTEREST ON AMOUNT OWING) TO FLAT RATE

93

THAT
$40000 LOAN @ 14% P.A., 25 YEARS, $482 P.M.

PAYMENTS PRINCIPAL
25 X 12 X 482 — 40000 = ...

DIVIDE BY 25 = THIS ... (INTEREST FOR 1 YEAR)

DIVIDE BY 400 = 10.46 FLAT RATE
TH.AT

REMEMBER: THIS / OF TH.AT %

REDUCIBLE TO FLAT

$12000 LOAN @ 13% REDUCIBLE

7 YEARS $18.19 P.M. PER $1000

TOTAL INTEREST 12X18.19X84—12000=...

DIVIDE BY 7 =... INTEREST PER ANNUM

DIVIDE BY 120 ÷ 7.54% FLAT (7 YEARS ON 12000)

$635 ITEM. 20% DEPOSIT = $\frac{1}{5}$ x 635 = $127

BALANCE ($508) @ 8.5% FLAT P.A. 2 YEARS

($508 = 100%)

TOTAL PAYMENT 127 + 1.17 X 508 = $721.36

TOTAL INSTALMENTS − 127 = $594.36

DIVIDE BY 24 = $24.77 PER MONTH

INTEREST PAID .17 X 508 = $86.36 (594.36 − 508)

FINANCE: TASK 1.

85 FROM 9% TO 12%
LOAN X
INCREASE $60 PA

86 $18000 CASH
OR 25% DEPOSIT + 20 X $700 MONTHS

87 $20 PER HOUR: 30 NORMAL, 7 DOUBLE.
5 "TIME-AND-A-HALF"

89 $18 FIRST 8, 1½ NEXT 2,
THEN DOUBLE >10: CALCULATE 12 HOURS

$28000 PA
4 WEEKS + 18%

90 $18 + 15% P.A
8 HOURS

$16 PH: SAT: 4@ 1½
7 HOURS
THEN DOUBLE:
SUN: DOUBLE

91 3000 @ 12 CENTS

4% INCREASE ON $700 PW
DIFFERENCE PA

92 $200 RETAINER P.W.
+ 8% ON $3000

DISCOUNT: 15% + 3% IF
PAINT $400

93 $50 000 @ 14% P.A. 25 YEARS, $500 P.M.

94 $15000 @ 14% P.A. 8 YEARS. $20 P.M. PER $1000

95 $700, 20% DEPOSIT. BALANCE @ 8% FLAT RATE P.A.
2 YEARS

100% — 84%

LOAN $8123 6 YEARS 14% P.A.

REPAYMENTS 8123 X 1.84 ÷ 72 = $207.61 P.M.

AS A DECIMAL

VARIATION 1.

$8123, 6Y, $207.61 P.M. X% P.A.

(207.6 PAYMENTS X 72 − 8123 PRINCIPAL) ÷ 6 = ... ÷ 81.23 = 14% P.A.

VARIATION 2.

$8123, 14% P.A. $217.61 P.M. X YEARS

$$217.6 \times 12x - .14x \times 8123 = 8123x$$

PAYMENTS — INTEREST — PRINCIPAL

$$x = 8123 \div (217.6 \times 12 - .14 \times 8123) = 6$$

VARIATION 3.

$217.61 P.M. 6 YEARS 14% P.A. $x (PRINCIPAL)

PRINCIPAL + INTERST = PRINCIPAL + INTEREST

$$1.84x = 217.6 \times 72 \therefore x = 8123$$

COMPARING PRICES

365.

66.

A SIMPLE RECIPE

MONEY ÷ QUANTITY = MONEY

IGNORE DECIMAL POINTS

1 KILO	$4.70	470 ÷ 1000	.47 MENTALLY
300 g	$1.38	138 ÷ 300	.46 CHEAPER OR BEST BUY
750 g	$3.60	36 ÷ 75	.48
$2\frac{1}{2}$ kg	$12.25	1225 ÷ 2500	.49

BANK CARD

18% P.A. = 1.5% P.M. + .14931% PER DAY AFTER 25 DAYS

CLOSING BALANCE $156.80

30 DAYS: NO PAYMENTS MADE

OPENING BALANCE $(1.015 + 5 \times .0014931) \times 156.8 =$

$159.54

100

LOAN $17300, 10.1% FLAT P.A., 10 YEARS

MONTHLY INSTALMENTS 201×173 ÷120 = $289.78

100%+101%

101

LOAN $1000. 7.75% FLAT P.A. 18 MONTHS

MONTHLY PAYMENTS (1000 + 77.5×1.5) ÷18 = $62.01

10×7.75

MONTHLY REPAYMENT TABLE

MONTHLY PER $1000

368.

% P.A.	2 YEARS $	5 YEARS $	7 YEARS $
12	47.07	22.24	17.65
12.5	47.31	22.50	17.92
13	47.54	22.75	18.19

$ 12000

13% P.A. 7 YEARS

PAYMENTS PER MONTH

12 X 18.19 = $218.28

BUILDING SOCIETY: $2500, 3 MONTHS

8% P.A. AT CALL

$50 INTEREST

8.5% P.A. 3-6 MONTHS

8.5 x 25 ÷ 4 = $53.13

(BEFORE TAX) **INCOME COMPARISON** 369

		PER ANNUM	
$ 385	WEEK	385 X 52	= $20120
$ 778	FORTNIGHT	778 X 26	= $20228
$ 1685	CALENDAR MONTH	1685 X 12	= $20220
$ 20152	YEAR		$20152

NOT PER DAY!

INFLATION RATE 3%

THIS MEANS THAT A $28500 CAR WILL COST

$28500 \times 1.03^{5} \doteqdot \$33000$ IN 5 YEARS TIME

NEAREST 100

APPRECIATION 6% P.A.

INCREASE IN PRICE

A $85000 BLOCK OF LAND WILL BE WORTH

$85000 \times 1.06^{5} \doteqdot \$113700$ IN 5 YEARS TIME

NEAREST 100

INCREASE IN PRICE RATIO 21:29

DIVIDE

IF $\frac{29}{21} = \frac{376}{x \div}$ THEN 21 TIMES OLD PRICE $ 259

NEW PRICE $ 376

105

REVISION

WHAT % IS 46 OF 828

$$\frac{46}{8.28} = 5.5\dot{} \%$$

INCREASE $8 BY 30%

100% + 30% = 130% = 1.3

1.3 X 8 = $10.40

DECREASE $8 BY 30%

100% − 30% = 70% = .7

.7 X 8 = $5.60

COST $128. 5% DISCOUNT, 10% G.S.T.

PAY 128 X .95 X 1.1 = $133.76

COST INCLUDING 15% DISCOUNT $ 80

READ →

← DO

MARKED PRICE 80 ÷ .85 = $94.12

FINANCE: TASK 2

97 LOAN $9000, 5 YEARS @ 18% P.A.
CALCULATE REPAYMENTS P.M.

$9000, 5 YEARS, $220 P.M. @ x%

98 $9000, 14% P.A. $220 P.M. x YEARS

$200 P.M. 6 YEARS, @ 18% P.A. PRINCIPAL x

99

2 KILOS	350 g	650 g	1 KILO
$8	$1.50	$3	$4.50

100 CLOSING BALANCE $170, SAME DETAILS
CALCULATE NEW (OPENING BALANCE

101 LOAN $18000, 9% FLAT RATE P.A. 5 YEARS
CALCULATE MONTHLY INSTALMENTS

$2000 @ 8% FLAT RA. 24 MONTHS

104 INFLATION RATE 4%. IN 4 YEARS.
A $16000 CAR WILL COST $x

FINANCE: TASK 3

104

APPRECIATON 8% P.A.

PROPERTY: $120 000. IN 6 YEARS TIME $...

105

INCREASE 10 : 14 NEW PRICE $500

OLD PRICE $X

WHAT % IS 56 OF 280?

106

INCREASE 24 BY 12%

DECREASE 48 BY 14%

107

COST $234, 15% DISCOUNT. 10% GST

COST INCLUDING 5% DISCOUNT $120

PARABOLAS

744.

$x^2 + 2x - 3$ — QUADRATIC EXPRESSION

FACTORISED $(x+3)(x-1)$

$x^2 + 2x - 3 = 0$ — QUADRATIC EQUATION

$(x+3)(x-1) = 0$

x-VALUES ARE CALLED

$x = -3$ & $x = 1$

ROOTS OR ZEROS

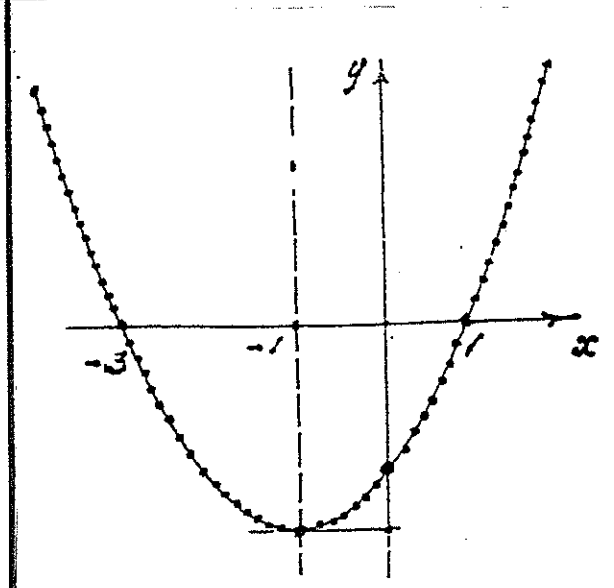
$y = x^2 + 2x - 3$ — PARABOLA

WHEN $y = 0$, -3 & 1 ARE THE ROOTS, ZEROS OR x-INTERCEPTS

AXIS OF SYMMETRY $x = -1$ (AVERAGE OF -3 & 1)

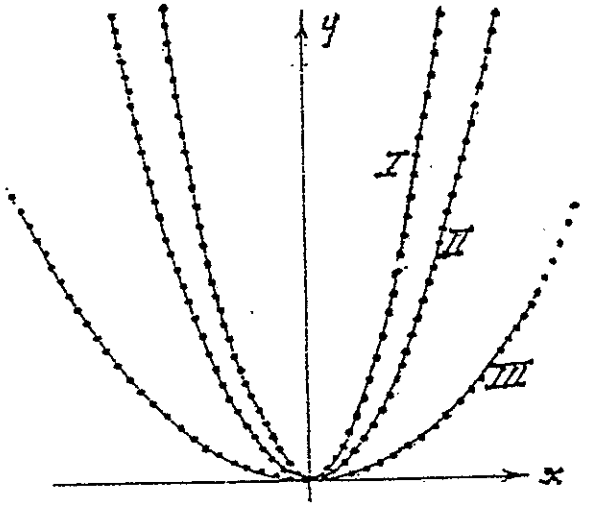
SUBSTITUTE $\therefore$ VERTEX $(-1, -4)$. y-INTERCEPT $(0, -3)$

MINIMUM



STEEP	STANDARD PARABOLA	SHALLOW
$y = 2x^2$	$y = x^2$	$y = \frac{1}{2}x^2$
POSITIVE CO-EFFICIENT		
CONCAVE UP	CONCAVE UP	CONCAVE UP

THE CO-EFFICIENT OF x^2 INFLUENCES THE SHAPE



WHEN $x = 0$, $y = 0$

EQUAL, REAL ROOTS

FOR ALL OTHER VALUES OF x, $y > 0$

746.

$y = x^2 + 4x + 4$

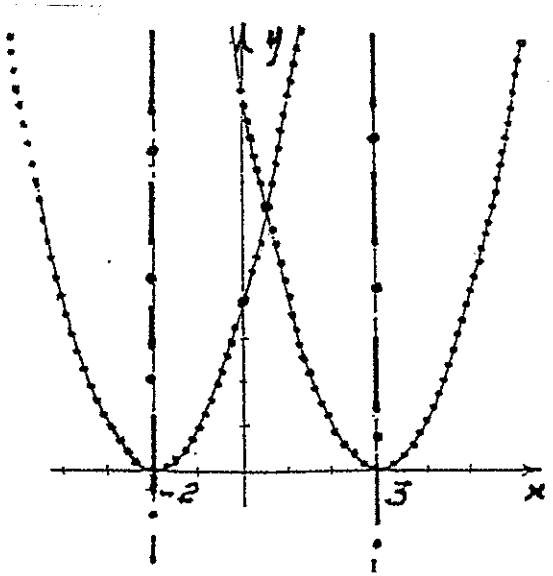
$y = (x+2)^2$

CONCAVE UP

EQUAL ROOTS $x = -2$

℄

AXIS



$y = x^2 - 6x + 9$

$y = (x-3)^2$

CONCAVE UP

EQUAL ROOTS $x = 3$

℄

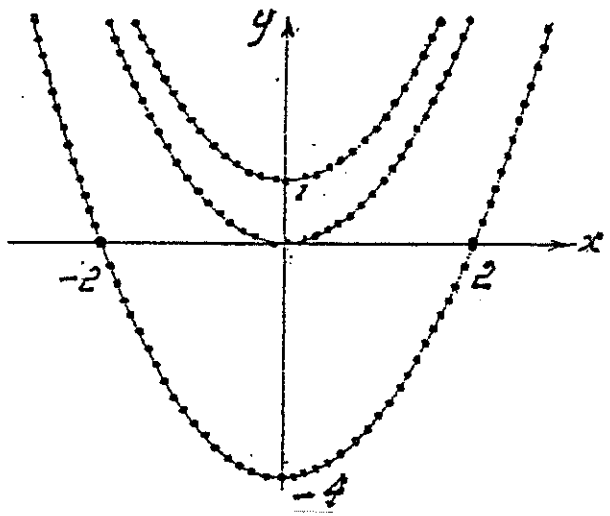
AXIS

POSITIVE CO-EFFICIENT of x^2 | PARABOLA CONCAVE UP

THREE POSSIBILITIES

$y = x^2 + 1$ NO REAL ROOTS

y IS POSITIVE DEFINITE



$y = x^2$ EQUAL REAL ROOTS $x = 0$

$y \geq 0$

$y = x^2 - 4 = (x-2)(x+2)$ 2 REAL ROOTS $x = \pm 2$

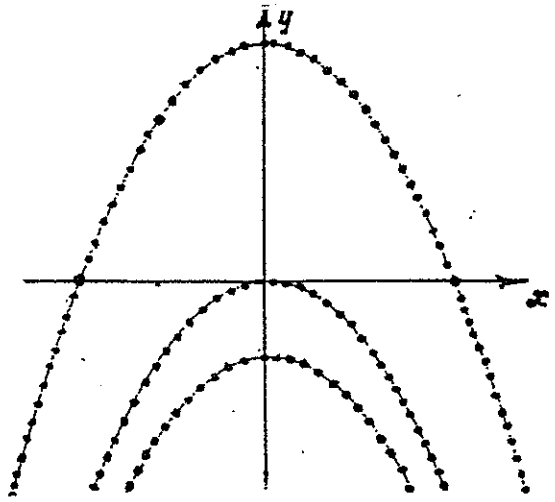
y IS INDEFINITE NEGATIVE, ZERO, POSITIVE

NEGATIVE COEFFICIENT OF x^2	PARABOLA CONCAVE DOWN	748.

THREE POSSIBILITIES

$y = -x^2 - 1$ NO REAL ROOTS

y IS NEGATIVE DEFINITE



$y = -x^2$ EQUAL, REAL ROOTS $x=0$

$y \leqslant 1$

$y = 4 - x^2 = -(x-2)(x+2)$ 2 REAL ROOTS $x = \pm 2$

y IS INDEFINITE

THE QUADRATIC FORMULA

749.

FOR THE GENERAL QUADRATIC EQUATION & PARABOLA

$$y = ax^2 + bx + c = 0$$

VISUALISE!

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

ALL CLEAR ON CALCULATOR

AXIS of SYMMETRY

A WAY TO REMEMBER

$b^2 - 4ac$ IS THE DISCRIMINANT Δ GREEK CAPITAL D; IT DISCRIMINATES

$\Delta < 0$	$\Delta = 0$	$\Delta > 0$
NO REAL ROOTS	EQUAL, REAL ROOTS	2 REAL ROOTS

S11

SPECIFIC PARABOLAS

750.

IF THE QUADRATIC EQUATIONS ($y=0$) CAN'T BE READILY FACTORISED, THE ALL-PURPOSE FORMULA IS USED TO FIND THE ROOTS (IF ANY).

HOWEVER, INSTEAD OF THE PRONUMERALS, THE SPECIFIC NUMERALS ARE USED **DIRECTLY!**

$y = 2x^2 - 3x - 7$

$\triangle > 0 \therefore$ 2 REAL ROOTS

$$x = \frac{3 \pm \sqrt{9+56}}{4}$$

(PP. ; SQUARE)

$x \doteqdot 2.8$

$x = -1.3$

EQUATION AXIS $x = \frac{3}{4}$ SUB.

VERTEX $\left(\frac{3}{4}, -8\frac{1}{8}\right)$

$y>0$, $x<-1.3$ & $x>2.8$	$y=0$, $x=-1.3$ & 2.8	$y<0$, $-1.3<x<2.8$

911

A SPECIFIC PARABOLA

751.

WITHOUT FORMULA

$y = 3x^2 - 11x + 8$

$= (3x-4)(x-2)$

ROOTS 2 & $1\frac{1}{3}$ AXIS $x = 1\frac{2}{3}$, SUB.

VERTEX $(1\frac{2}{3}, -\frac{1}{3})$

FORMULA

$$x = \frac{11 \pm \sqrt{121-96}}{6}$$

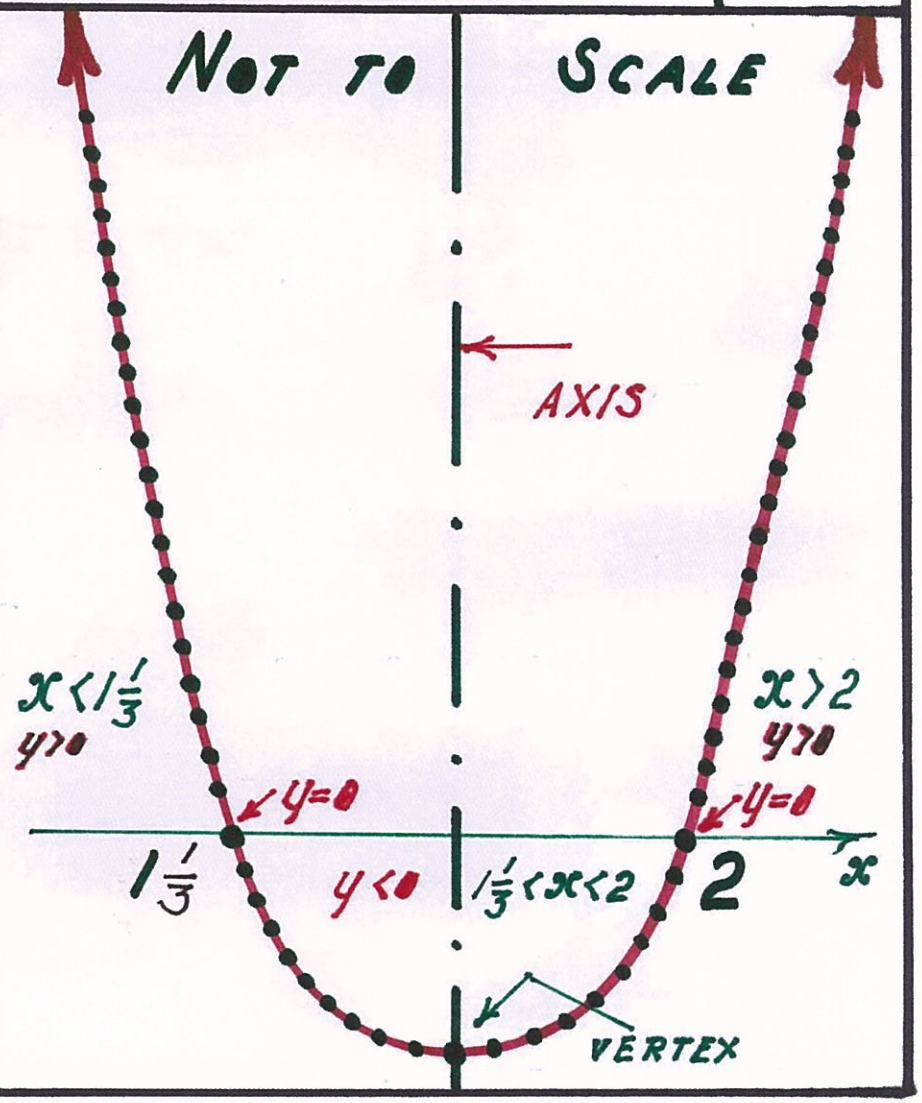
ROOTS

$x = 2$

$x = 1\frac{1}{3}$

AXIS $x = \frac{10}{6} = 1\frac{2}{3}$

ONLY WAY IF THERE ARE NO ROOTS



$$y = -3x^2 - 12x - 7$$

PARABOLA. CONCAVE DOWN ∴ THERE IS A MAXIMUM (y-VALUE)

118

144 − 84
DISCRIMINANT > 0

∴

2 REAL ROOTS

1. AXIS: $x = \frac{12}{2x-3} = -2$, SUB. $y = 5$

VERTEX $(-2, 5)$

USING DIFFERENTIATION 2. AT THE VERTEX, THE DERIVATIVE OF y = THE DIFFERENTIAL = CHANGE

y' = THE GRADIENT m OF THE TANGENT $= -6x - 12 = 0$ ∴ $x = -2$, $y = 5$

SINCE THE 2ND DERIVATIVE $y'' = -6$ (< 0), $y = 5$ IS A MAXIMUM.


OAKHILL

PARABOLA: SUMMARY & TASK

THE LOOK: $y = 2x^2 + 4x + 5$

THE SHAPES: 111

6 POSSIBILITIES: 113, 114

THE PLOTTING: INVENT X-VALUES TO CALCULATE Y

(x, y). USE A FLEXI RULER TO JOIN THE POINTS.

STANDARD CALCULATIONS:

THE DISCRIMINANT Δ: $(b^2 - 4ac)$ 115

AXIS OF SYMMETRY: $x = \frac{-b}{2a}$ 115, 118

118

THE VERTEX: 1. SUBSTITUTE THE ABOVE TO FIND Y

THERE IS A MAXIMUM OR A MINIMUM

OR USE 2. DIFFERENTIATION

	DISCOVER THE PATTERN	
$y = 3x^2 + 5x$	$y' = 6x^1 + 5$	$y'' = 6$

QUADRATIC EQUATIONS

857.

$$4(x^2 - 7) = 8$$

$$x^2 = 9$$

$$x = \pm 3$$

$$x(4 - x) = 1$$

$$x = 1 \qquad x = 4$$

858.

$$3x(1-4x)=0$$

$$x=0 \quad , \quad x=\frac{1}{4}$$

TYPE 1.

$$(x-2)(x+5)=0$$

THINK OPPOSITES

$$x=2 \quad x=-5$$

TYPE 2.

859.

$$(5x-2)(3x+3)=0$$

DIVIDE BY — OPPOSITE

$$x=\frac{2}{5} \qquad x=-1$$

$$(6x-12)(6x+3)=0$$

$$x=2 \quad , \quad x=-\frac{1}{2}$$

FACTORISING BINOMIALS

NAME

861.

$x^2 - 6x = x(x-6) = 0$

1 SET

MEANS $x \times ($

2 NAMES

$x = 0 \quad , \quad x = 6$

$2x^2 - 6x = 2x(x-3) = 0$

$x = 0 \quad , \quad x = 3$

$3x^2 - 12 = 3(x^2 - 4) = 0$

$x = \pm 2$

FACTORISING TRINOMIALS

NAME

861.

TYPE 1: $A=1$ (NOT WRITTEN)

$$x^2 - x - 12 = 0$$

END PRODUCT:
FIND 2 FACTORS,
1 NEG. & 1 POS.

NEGATIVES 'WIN' BY 1

SET ONE SET TWO

$$(x-4)(x+3)=0$$

THINK OPPOSITE

$$x=4 \quad , \quad x=-3$$

$$x^2 - 7x + 6 = 0$$

SET IT UP IN STAGES

LOOKS LIKE $(\ \text{SET 1}\ )(\ \text{SET 2}\ ) = 0$

INSERT SIGNS $(x - \ )(x - \ ) = 0$

ONLY WRITE THIS OF COURSE $(x - 1)(x - 6) = 0$

OPPOSITE

$x = 1$ $\qquad$ $x = 6$

ONLY USE FORMULA!

TYPE 2
A PRIME

863.

$$2x^2 - 5x - 12 = 0$$

THE CROSS-MULTIPLY WAY

TRIAL & ERROR

$$(2x + 3)(x - 4)$$

$-8x + 3x = -5x$

END PRODUCT

END PRODUCT

$$x = -\frac{3}{2}, \quad x = 4$$

864.

À PRIME $5x^2 - 11x + 2 = 0$

$(5x - 1)$

$(x - 2)$

$x = \frac{1}{5}$, $x = 2$

TYPE 3: A COMPOSITE

865.

MORE POSSIBILITIES

$$6x^2 + 11x - 4 = 1$$

$$(3x \quad -1)$$

$$(2x \quad +4)$$

$$x = \frac{1}{3} \quad , \quad x = -2$$

128

866.

$$x + \frac{2}{x} = 3$$

$$x^2 - 3x + 2 = (x-2)(x-1) = 0$$

$x = 2$, $x = 1$

$$\frac{4}{x} - x = 3$$

$$x^2 + 3x - 4 = (x+4)(x-1) = 0$$

$x = -4$, $x = 1$

COMPLETING THE SQUARE

$$x^2+6x+C$$

HALVE & SQUARE →

$$x^2+6x+9 = (x+3)^2$$

$$x^2-Bx+49 \qquad B>0$$

← DOUBLE & SQUARE ROOT

$$x^2-14x+49 = (x-7)^2$$

$B=14$

871.

$$x^2 - 2x - 8 = 1$$

SOLUTION 1.

$$(x-4)(x+2) = 1 \qquad x = 4, \quad x = -2$$

SOLUTION 2. USING 'COMPLETING THE SQUARE'

REWRITE AS

$$x^2 - 2x + 1 = 9$$

$$(x-1)^2 = 9 \qquad x = 4, \quad x = -2$$

$$x - 1 = \pm 3$$

COMPLETING THE SQUARE 2.

$$9x^2 + 6x = 4$$

REWRITE AS

$$x^2 + \frac{2}{3}x + \frac{1}{9} = \frac{5}{9}$$

$$\left(x + \frac{1}{3}\right)^2 = \frac{5}{9} \quad . \quad \left(x + \frac{1}{3}\right) = \pm\frac{1}{3}\sqrt{5}$$

$$x = \frac{1}{3}\left(-1 \pm \sqrt{5}\right)$$

872.

WRITE $3x^2 - 2x + 5 = 1$ (A)

IN THE FORM $A(x+1)^2 + B(x-2) + C = 1$

$A(x^2 + 2x + 1) + Bx - 2B + C = 1$

$= Ax^2 + 2Ax + A$ ↑ COMBINE

$Ax^2 + (2A+B)x + (A - 2B + C) = 1$

5

$A = 3;\ (2A+B) = -2 \therefore B = -8,\ C = -14$

$3(x+1)^2 - 8(x-2) - 14 = 1$

ALL PURPOSE FORMULA

873.

BASED ON THE GENERAL FORM

$$ax^2+bx+c=1$$

134

VISUALISE !
NEVER WRITE DOWN

ALL CLEAR

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

1 2 3

2xa
2xb
1xc

X

VISUALISE TO REMBER

LOOK FOR SIMILARITIES

$$2x^2 - 3x - 7 = 1$$

ROOTS IN SURD FORM (MEANS USE FORMULA)

APPLY DATA DIRECTLY

$$x = \frac{3 \pm \sqrt{9+56}}{4}$$

A WAY TO REMEMBER

$$x = \frac{3 \pm \sqrt{65}}{4}$$

875.

$3x^2 - 11x + 8 = 1$

$x = \frac{11 \pm \sqrt{121 - 96}}{6}$

$x = 2 \quad , \quad x = 1\frac{1}{3}$

$2x^2 + 2x - 3 = 1$

$x = \frac{-2 \pm \sqrt{4 + 24}}{4}$

$x = \frac{-1 \pm \sqrt{7}}{2}$

$3x^2 - x - 3 = 1$

$x = \frac{1 \pm \sqrt{1 + 36}}{6}$

$x = \frac{1 \pm \sqrt{37}}{6}$

GRAPHIC SOLUTION

876.

$$x^2 - 2x - 3 = 1$$

131

REWRITE AS

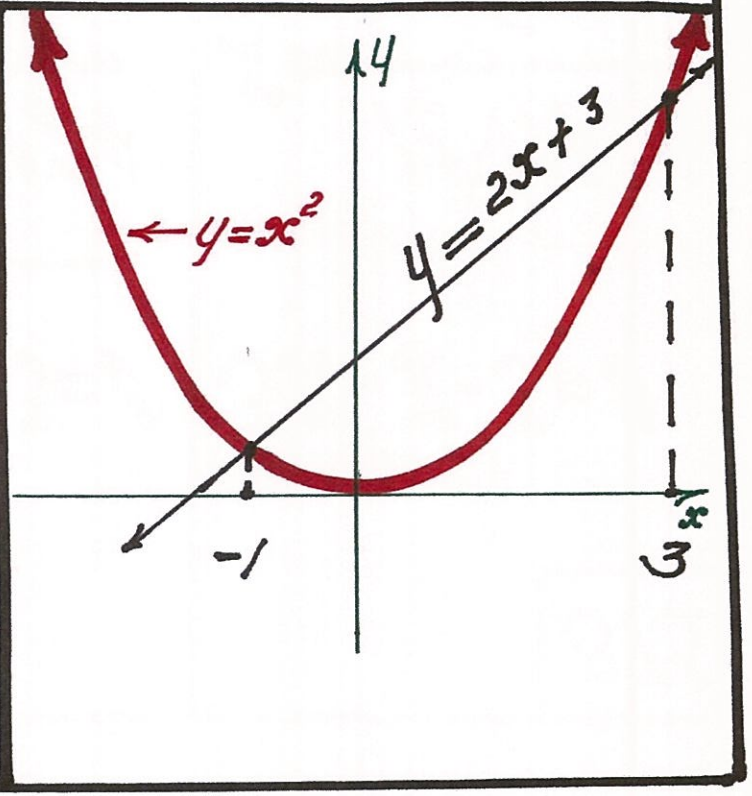
$y = x^2 = 2x + 3$

PARABOLA — STRAIGHT LINE

THE ROOTS ARE THE x-INTERCEPTS

CHECK: $(x-3)(x+1) = 1$

$x = 3, \quad x = -1$



FROM ROOTS TO EQUATION

877.

$x = -2$

α ALPHA

$x = 3$

β BETA (ALPHABET)!

(ALPHA

$(x+2) \times (x-3) = 0$

1

2

3

4

COMBINE STEPS 2 & 3 MENTALLY

$x^2 - x - 6 = 0$

139

RELATIONSHIP BETWEEN

878.

ROOTS α (alpha) & β (beta) AND a, b, c

OF THE GENERAL FORM $ax^2+bx+c=1$

$$(x-\alpha)(x-\beta)=x^2-(\alpha+\beta)x+\alpha\beta=1$$

$\equiv$ IDENTICAL TO

$$x^2+\frac{b}{a}+\frac{c}{a}=1$$

$$\alpha+\beta=\frac{-b}{a}$$

$$\alpha\beta=\frac{c}{a}$$

139

879.

$$2x^2 - 5x + 1 = 1$$

$$\alpha + \beta = \frac{5}{2} \qquad \alpha\beta = \frac{1}{2}$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{5}{2} \div \frac{1}{2} = 5$$

$$(\alpha+1)(\beta+1) = \alpha\beta + \alpha + \beta + 1 =$$

$$\frac{1}{2} + \frac{5}{2} + 1 = 4$$

$$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta =$$

$$\frac{25}{4} - 1 = 5\frac{1}{4}$$

881.

$$2x^2 - kx + 5 = 1$$

$$\alpha = 1$$

$$\alpha + \beta = \frac{k}{2} \therefore \beta = \frac{k}{2} - 1$$

$$\alpha\beta = \frac{5}{2} \quad \beta = \frac{k}{2} - 1 \therefore k = 7$$

THE DISCRIMINANT

$$\Delta = b^2 - 4ac$$

GREEK CAPITAL: DELTA

IT DISCRIMINATES:

IT SHOWS WHETHER $ax^2 + bx + c = 0$ HAS

2 DIFFERENT, REAL ROOTS	$\Delta > 0$
2 EQUAL, REAL ROOTS	$\Delta = 0$
NO REAL ROOTS	$\Delta < 0$

882.

$3x^2 - 11x + 8 = 0$

$\Delta = 121 - 96 \therefore$ 2 REAL ROOTS

$\Delta > 0$

$x^2 - 6x + 9 = 0$

$\Delta = 36 - 36 = 0 \therefore$ EQUAL ROOTS

$-2x^2 + x - 5 = 0$

$\Delta = 1 - 40 \therefore$ NO REAL ROOTS

$\Delta < 0$

QUADRATIC EQUATIONS: TASK 1.

120 $5(x^2-9)=0$

$x(12+x)=0$

121 $(4x-3)(7x+2)=0$

122 $(4x-8)(9x+3)=0$

123 $2x^2-8x=0$

$4x^2-16=0$

124 $x^2-3x-10=0$

$x^2-5x+6=0$

125 | 126 | 127 | 128

DIRECT METHOD WILL ALL PURPOSE FORMULA

SAME QUESTIONS, COMPARE YOUR ANSWERS!

129 $x-\frac{7}{x}=-12$ FORMULA

$x+\frac{5}{x}=14$ FORMULA

130 $x^2+10x+C$

$x^2-\beta x+64$

131 USE THE ALL PURPOSE FORMULA AS A ROUTINE

132 CHECK THE GIVEN RESULTS

133 WRITE $5x^2-3x+7=0$

IN THE FORM $A(x+2)^2+\beta(x-1)+C=0$

136 WITH FORMULA

137 $x^2+3x-10=0$

QUADRATIC EQUATIONS: TASK 2

138

$x = 4$ & $x = -5$ | $x = -3$ & $x = 7$

1. $(x - \alpha)(x - \beta) = 0$

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

GENERAL FORM

$ax^2 + bx + c = 0$

DIVIDE BY a

$x^2 + \frac{b}{a} + \frac{c}{a}$

IN SIMILAR POSITIONS

$\therefore \quad \alpha + \beta = -\frac{b}{a}$ & $\alpha\beta = \frac{c}{a}$

140

$3x^2 - 6x + 2 = 0$

141

$4x^2 + kx + 7 = 0, \quad \alpha = 2$

143

ANALYSE

$x^2 + 2x - 3 = 0$

ANALYSE

$x^2 + 4x + 4 = 0$

ANALYSE

$-x^2 - 1 = 0$

SEQUENCES & SERIES

941.

ARITHMETIC PROGRESSION

IN $8, 11, 12, \ldots$ $a = T_1 = 8$ (TERM ONE), $d = 2$ (difference) — 1.

IN $15, 12, 9, \ldots$ $a = 15$, $d = -3$ — 2.

ARITHMETIC MEAN TEST

1. $8 + 12 = 2 \times 11$

2. $\frac{15+9}{2} = 12$ BY DEFINITION

SEQUENCES & SERIES

ARITHMETIC PROGRESSION

PROVE $T_n = 3n - 2$ IS AN A.P.

$T_1 = 1,\ T_2 = 4,\ T_3 = 7$ | $\frac{1+7}{2} = 4$ ✓

$a = T_1 = 1,\quad l = \text{last} = T_{20} = 58$

$S_{20} = \text{SUM 20 TERMS} = \frac{20}{2}(1+58) = 590$

THE A.P. PATTERN

943.

$T_1 = a$ $\qquad$ $T_2 = a + 1d$ $\qquad$ $T_3 = a + 2d$ ETC

IN GENERAL $\quad T_n = a + (n-1)d$

nth TERM. $\qquad$ n IS THE number OF TERMS.

THE G.P. PATTERN

GEOMETRIC PROGRESSION LIKE 2, 4, 8, 16, 32 ...

$T_1 = a$ $\qquad$ $T_2 = ar^1$ $\qquad$ $T_3 = ar^2$

IN GENERAL $\quad T_n = ar^{n-1}$

TEST

$5, 8, 11, \ldots \quad \frac{5+11}{2} = 8 \therefore$ A.P. $d = 3, a = 5$

$T_{21} = 5 + 19 \times 3 = 62$

VISUALISE THE PATTERN;

APPLY NUMBERS DIRECTLY

-1

GP TEST $r = \frac{-18}{54} = \frac{-1}{3}$

$54, -18, 6, \ldots (-18)^2 = 6 \times 54 \therefore$ G.P.

$T_7 = 54\left(\frac{-1}{3}\right)^6 = \frac{54}{729} = \frac{2}{27}$

$3^6 = 729$

-1

945.

A.P. $T_n = 3n + 5$

SUB. $n=1$	SUB. $n=2$	SUB. $n=3$	SUB. $n=4$
$T_1 = 8$	$T_2 = 11$	$T_3 = 14$	$T_4 = 17$

G.P. $T_n = 2^n$

$T_1 = 2$	$T_2 = 4$	$T_3 = 8$	$T_4 = 16$

150

SIMULTANEOUS EQUATIONS

A.P. $T_5 = 31$, $T_{12} = 73$

$a + 11d = 73$ 1.

$-\ a + 4d = 31$ 2.

$7d = 42$ $\therefore d = 6$ SUB 2. $a = 7$

G.P. $T_3 = 3$ $T_{10} = 6561$

$ar^9 = 6561$ 1.

$ar^2 = 3$ 2.

DIVIDE

$r^7 = 2187$ PRESS x^y 1 $a^b/_c$ 7 $= 3$ $= r$ SUB 2

$a = \frac{1}{3}$

A.P. $7, 12, 17, \ldots$ $T_n = 372$

$7 + (n-1) \times 5 = 372$

$n - 1 = 73$

$n = 74$

number of terms

G.P. $3, 6, 12, \ldots$ $T_n = 384$

$3 \times 2^{n-1} = 384$

$2^{n-1} = 128$

$(n-1) \ln 2 = \ln 128, \quad (n-1) = \ln 128 \div \ln 2 = 7$

MENTALLY

$n = 8$

A.P. 3, 7, 11, ... LAST TERM > 211

$3 + (n-1) \times 4 > 211$ | $T_{51} > 211$

n = pos. INTEGER $\quad n > 197 \div 4 + 1 = 50.25 \quad \therefore$

153

G.P. $a = \frac{1}{2}$, $r = 3$, LAST TERM > 311

$\frac{1}{2} \times 3^{n-1} > 311 \quad \therefore T_7 > 311$

$3^{n-1} > 622 \quad$ Do $(n-1) \ln 3 > \ln 622$, $n-1 > \ln 622 \div \ln 3 > 5.8$

G.P.	GEOMETRIC PROGRESSION	942.

$2, 6, 18, \cdots$

$a = T_1 = 2$ ratio $r = 6 \div 2 = 3$

$16, 12, 9, \cdots$ $a = 16$, $r = \frac{12}{16} = \frac{3}{4}$

1. GEOMETRIC MEAN TEST 2.

$\sqrt{2 \times 18} = 6$ | $\sqrt{16 \times 9} = 12$

27, 9, 3, ... GEOMETRIC PROGRESSION

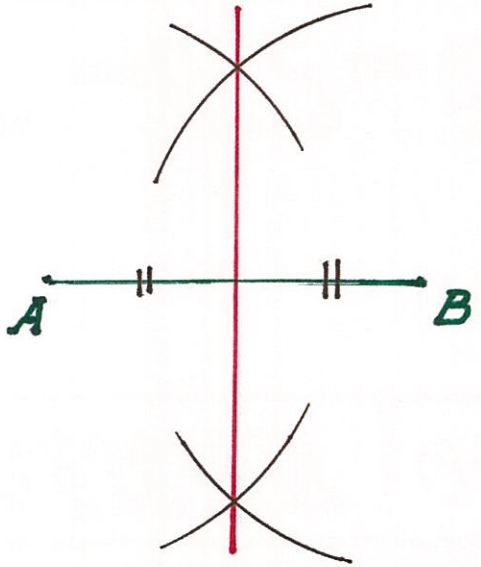
BECAUSE $9^2 = 3 \times 27$. $a = T_1 = 27$

ratio $r = \frac{3}{9} = \frac{1}{3} < |1|$ ∴ LIMITING SUM $= \frac{a}{1-r} = 40\frac{1}{2}$

LOCUS OF P, EQUIDISTANT FROM A & B,

PATH

IS THE PERPENDICULAR BISECTOR



SEQUENCES & SERIES: TASK

146 | 16, 20, 24 | $a=\ldots$, $d=\ldots$ (T_1) SHOW AP TEST

147 | PROVE $T_n = 4n-3$ IS AN AP

147 | $T_1 = 2$, $T_{30} = 60$. CALCULATE S_{30}

148 / 149 | AP 6, 10, 14, $\ldots T_{24}=\ldots$ | GP: $a=2$, $r=3$ (2, 6, 18, 54, $\ldots$ $T_5 = 2\times3^4 = \ldots$ SEE 154

150 | AP $T_n = 4n+3$, $T_7 = \ldots$ | GP $T_n = 4^n$, $T_5 = \ldots$ SEE 154

151 | AP $T_4 = 10$, $T_{10} = 28$ | GP $T_3 = 8$, $T_7 = 128$ SEE 154

152 | AP 6, 10, 14. $T_n = 30$, $n = \ldots$ | G.P 2, 6, 18, $\ldots$ $T_n = 486$ $n=$ SEE 154

153 | AP 3, 6, 9. LAST ONE > 130 | GP 2, 4, 8, $\ldots$ LAST > 176 SEE 154

154 | 3, 6, 12, $\ldots$ $T_1 =$ $r =$ → | TEST GP

155 | SHOW 4, 12, 36, $\ldots$ IS A GP

GROUPED FREQUENCY

APPROXIMATIONS FOR LARGE NUMBERS

1013.

CLASS	CENTRE x	FREQ f	SUB TOTAL fx
56 - 58	57	8	456
59 - 61	60	12	720
62 - 64	63	33	2079
65 - 67	66	31	2046
68 - 70	69	12	828
71 - 73	72	4	288
	Σ	100 DAYS	TOTAL 6417

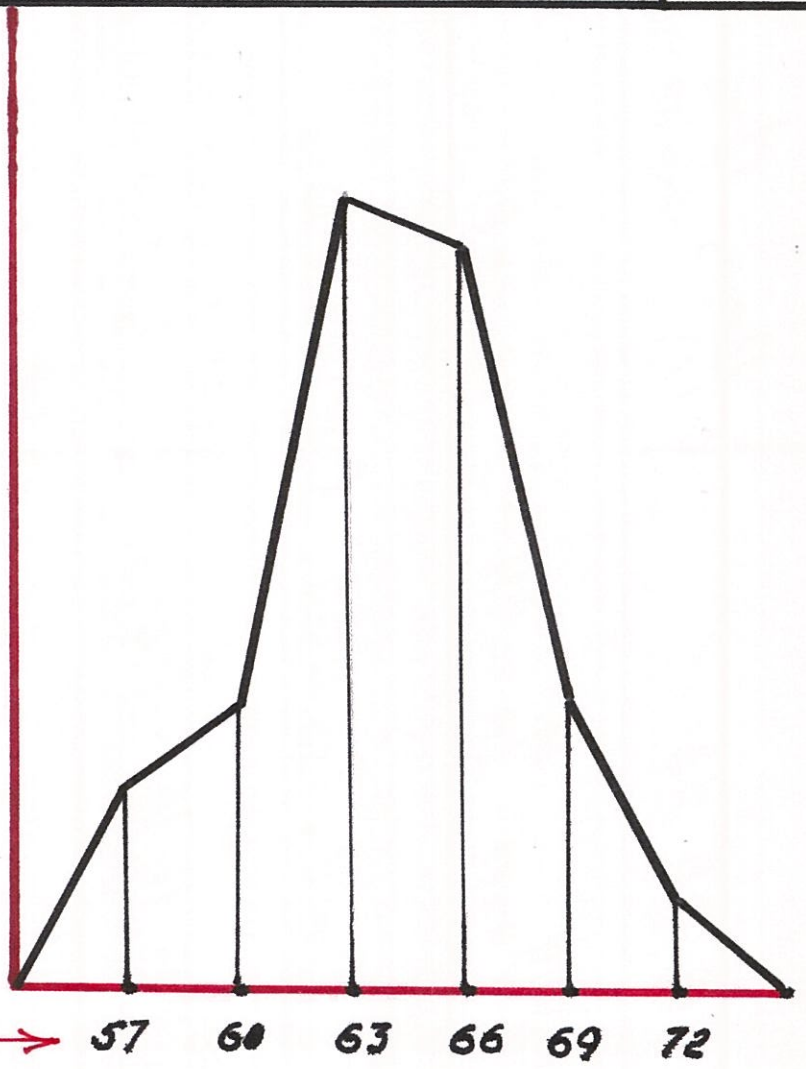
SOLD: $\bar{x} = 64.17$ PER DAY

REAL 64.08

REAL $\bar{x}$ 64.08

MODE 33 RANGE 17 (73-56)

CLASS INTERVAL 3



MEAN DEVIATION

THE AVERAGE OF THE DIFFERENCES (ABSOLUTE VALUES) BETWEEN SCORES & MEAN $\bar{x}$

1014.

MARKS OUT OF 10

HISTORY	FRENCH
9. 6. 8. 7. 8. 5. 6. 9. 8. 4.	10. 5. 9. 7. 9. 4. 5. 10. 9. 2.
MEAN ($\bar{x}$) = 7	MEAN ($\bar{x}$) = 7
MEAN DEVIATION (2+1+1+0+1+2+1+2+1+3) ÷ 10 = 1.4	MEAN DEVIATION (3+2+2+0+2+3+2+3.2.5) ÷ 10 2.4
SCORE 9 IN HISTORY IS MORE THAN 1 MEAN DEV. ABOVE $\bar{x}$	SCORE 9 IN FRENCH IS LESS THAN 1 M.D ABOVE $\bar{x}$ ∴

SCORE 9 IN HISTORY IS THE BEST RELATIVE MARK.

159

VARIANCE	STANDARD DEVIATION	1015.
AVERAGE of THE SQUARED DIFFERENCES	$\sqrt{\text{VARIANCE}}$	
SEE 1014	SEE 1014	
HISTORY	**HISTORY**	
$26 \div 10 = 2.6$	$\sqrt{2.6} = 1.6$ (δn)	
FRENCH	**FRENCH**	
$72 \div 10 = 7.2$	$\sqrt{7.2} = 2.7$ (δn) CALCULATOR	

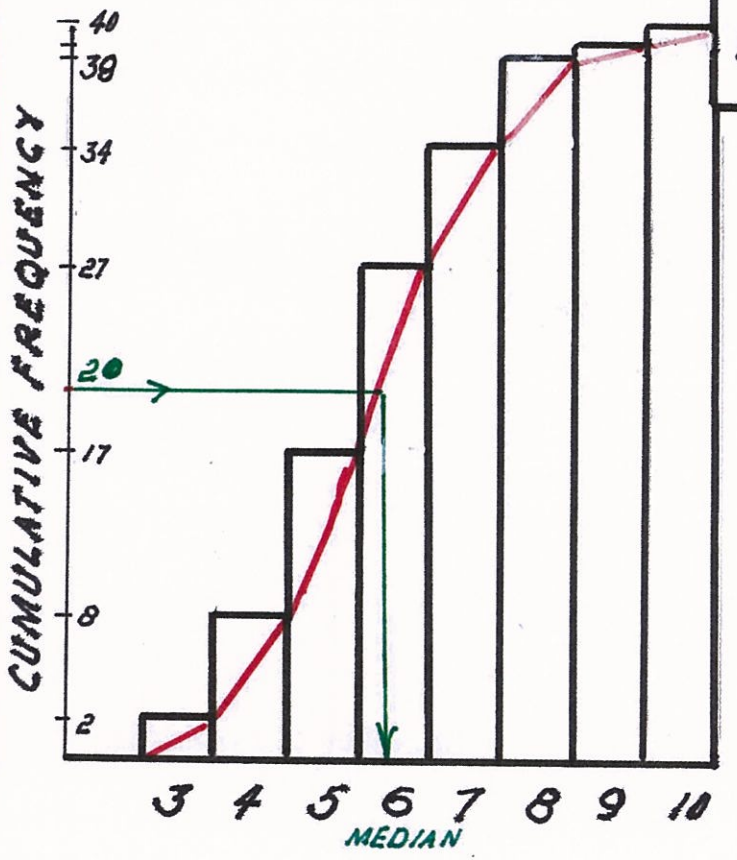
091

1016.

MARK	FREQUENCY	CUM. FREQ
3	2	2
4	6	= 8
5	9	17
6	10	27
7	7	34
8	4	38
9	1	39
10	1	40

27 STUDENTS SCORED 6 OR LESS

MEDIAN: THE 20TH & THE 21ST;
THEY ARE IN THE **FIRST** 27 ∴
MEDIAN = 6

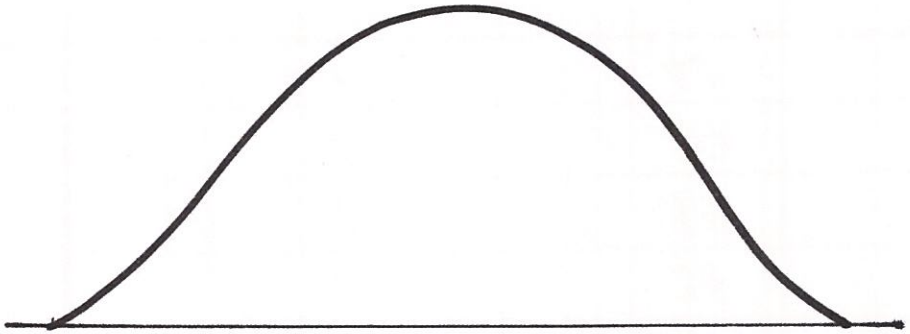


HISTOGRAM &
POLYGON (OGIVE)

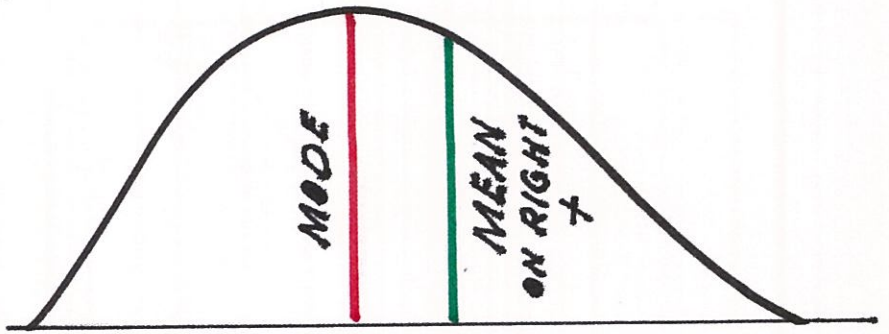
NORMAL & SKEWED DISTRIBUTIONS

1017.

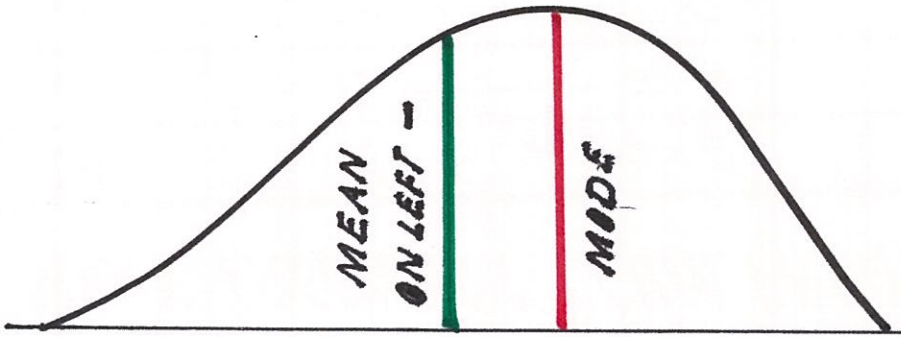
IF CLASS INTERVALS ARE SMALLER AND THE NUMBER OF MEASUREMENTS LARGER, THE FREQUENCY POLYGON APPROACES A CURVE.



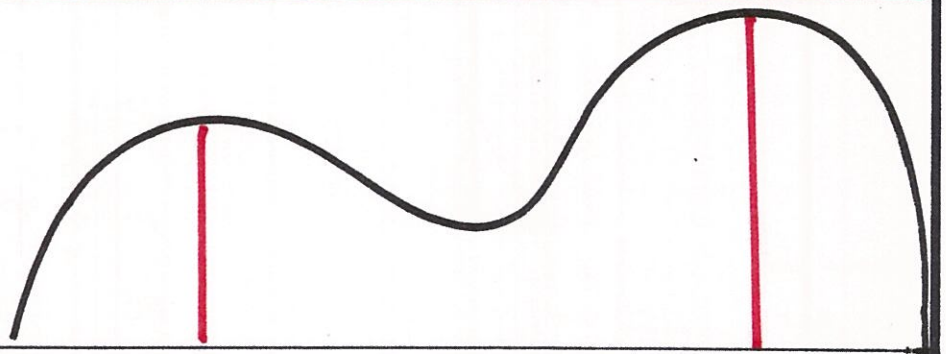
BELL SHAPED - NORMAL -



POSITIVELY SKEWED



NEGATIVELY SKEWED



BI-MODAL OR TWO-HUMPED

MEASURES of DISPERSION

1018.

VARIABILITY of DATA

MARKS FOR A		MARKS FOR B	
HIGHEST	78	HIGHEST	100
LOWEST	54	LOWEST	30
RANGE	24	RANGE	70
MEAN	65	MEAN	65
LESS VARIABLE		MORE VARIABLE	

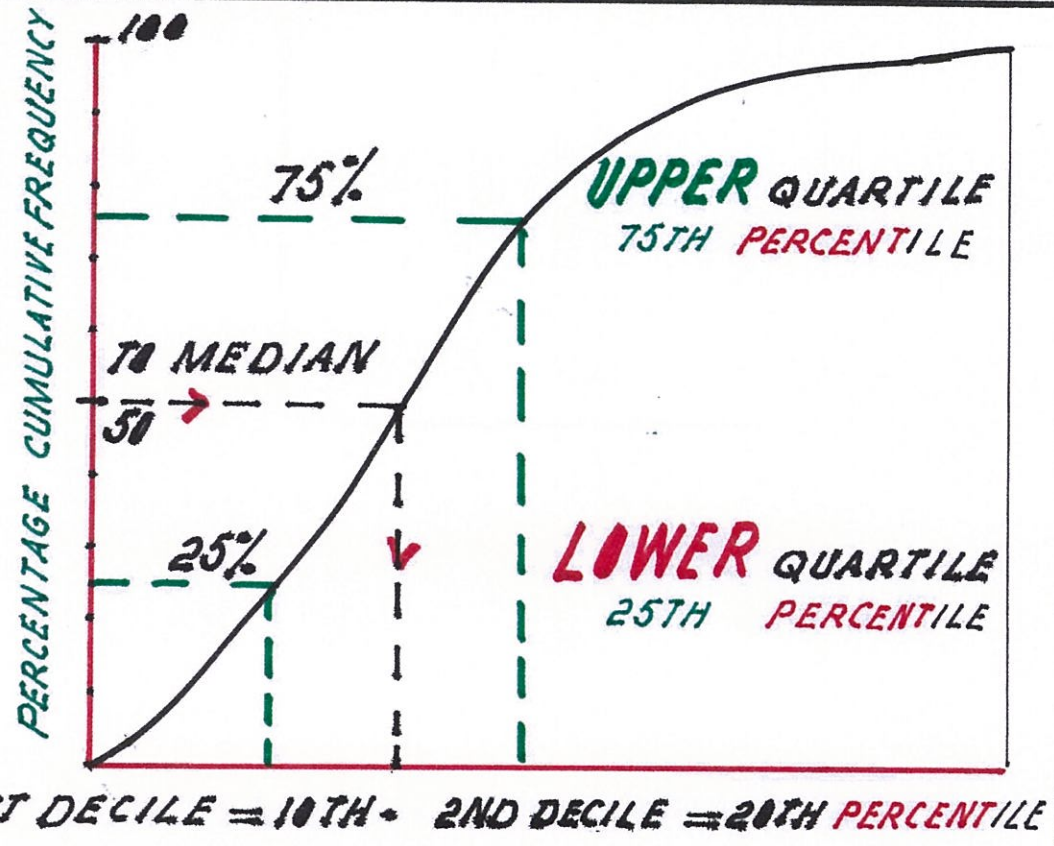
THE RANGE DOES NOT SHOW **THE SCATTER** FROM THE MEAN.

HOWEVER, IT'S USEFUL IN WEATHER REPORTING.

INTER QUARTILE RANGE

NOT AFFECTED BY EXTREMES

RANGE of SPREAD CONTAINING 50% OF THE SCORES



STANDARD DEVIATION LIMITS

1119.

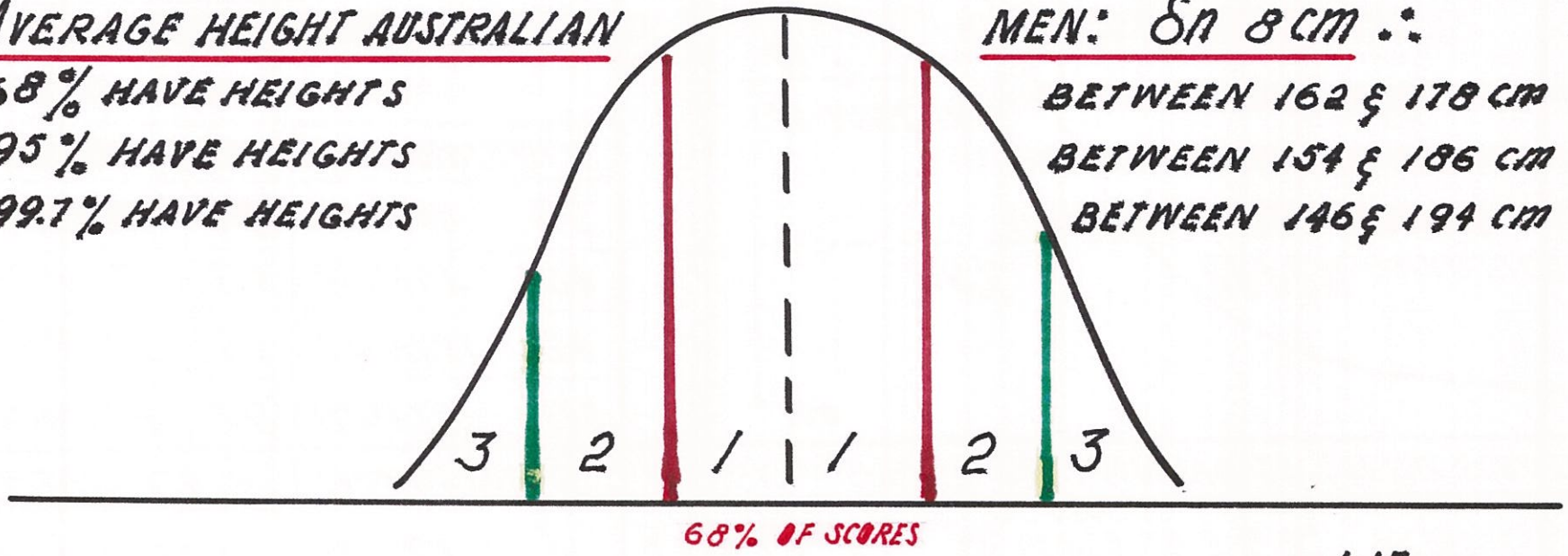
ONE S.D.	2 SIGMA	3 SIGMA

AVERAGE HEIGHT AUSTRALIAN MEN: $\bar{x}$ 8 cm $\therefore$

68% HAVE HEIGHTS BETWEEN 162 & 178 cm

95% HAVE HEIGHTS BETWEEN 154 & 186 cm

99.7% HAVE HEIGHTS BETWEEN 146 & 194 cm

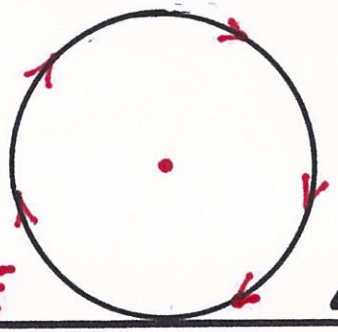


99.7% OF SCORES WILL ALMOST CERTAINLY LIE WITHIN 3 SIGMA

95% OF SCORES WILL VERY PROBABLY LIE WITHIN 2 SIGMA

SPHERE

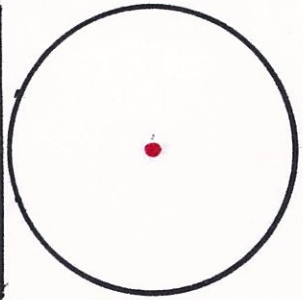
981.



CIRCUMFERENCE $2R\pi$

2R

SURFACE AREA SPHERE = S.A. FITTING CYLINDER WITHOUT TOP & BASE



$4R^2\pi\ u^2$

VOLUME: THE SUM OF AN INFINITE NUMBER OF PYRAMIDS. HEIGHT R

SINCE $V_{PYRAMID} = \frac{1}{3}$ BASE. HEIGHT.

$\therefore\ V_{SPHERE} = \frac{4}{3}R^3\pi\ u^3$

SPHERE: RADIUS 3 CM

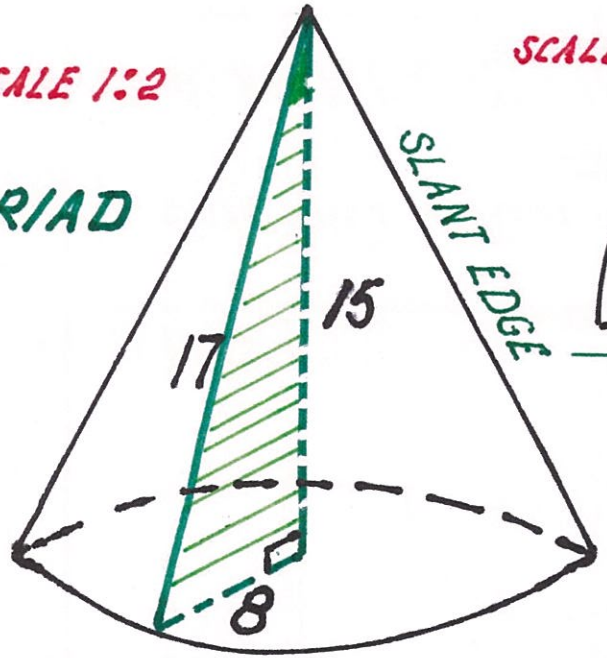
982.

S.A. $= 36\pi$ cm^2. VOLUME $= 36\pi$ cm^3

CONE: $R = 8$ cm, $H = 15$ cm. $V = 5 \times 64\pi = 320\pi$ cm^3

SCALE 1:2

TRIAD



SCALE 1:4

AREA $= 17 \times 8\pi = 136\pi$ cm^2

USE $\frac{1}{2}BH$ AS FOR TRIANGLE

$x = \frac{16}{34} \times 360^\circ \doteqdot 170^\circ$

TOTAL S.A $= 136\pi + 64\pi = 200\pi$ cm^2

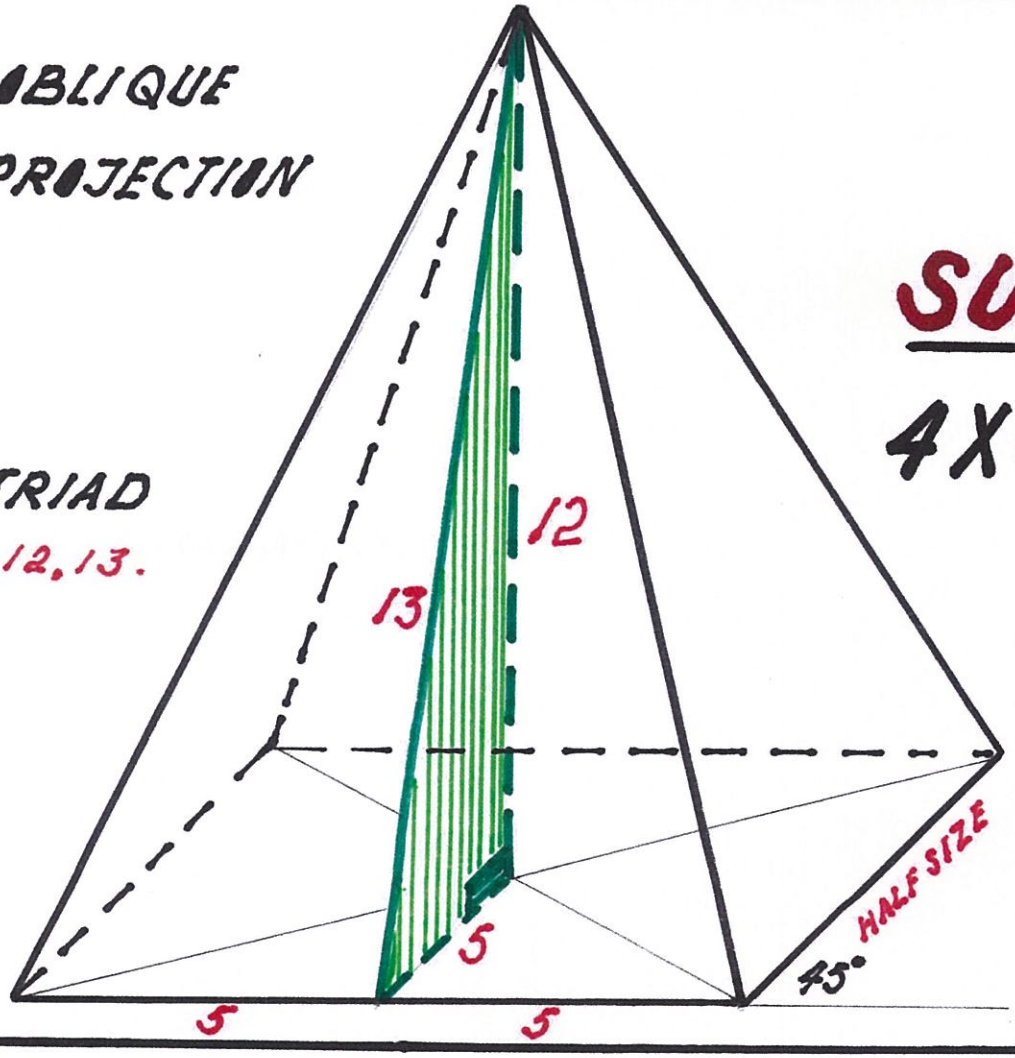
165

SQUARE PYRAMID

983.

991

OBLIQUE PROJECTION



TRIAD
5, 12, 13.

SURFACE AREA

$4 \times 65 + 100 = 360\ cm^2$

VOLUME

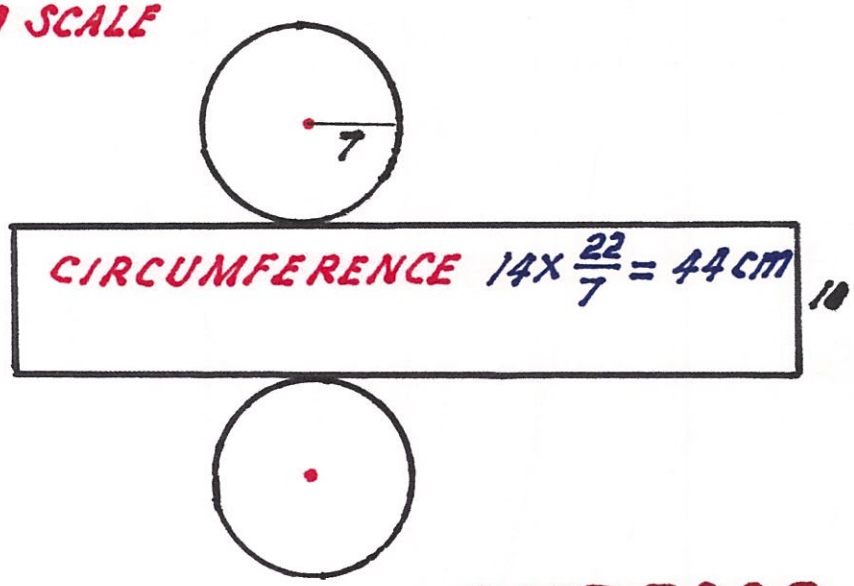
THINK ONLY: A THIRD BASE. HEIGHT

$400\ cm^3$

CYLINDER $R = 7_{cm}, H = 11_{cm}, \pi = \frac{22}{7}$ (USE)

984.

NOT TO SCALE



THINK ONLY (AREA) BASE . HEIGHT

$$V = 49 \times \frac{22}{7} \times 11 = 1541\,cm^3$$

SURFACE AREA

$$441 + 2 \times 49 \times \frac{22}{7} = 748\,cm^2$$

SIMILAR PRISMS

EDGES 1:2

SURFACE AREAS 1:4

VOLUMES 1:8

985.

WATER TANK $D = 4.7m, H = 1.4m$

$$V = 2.35^2 \times \pi \times 1.4 = 24.289\,m^3 = 24289\,L$$

65 mm

RAIN WATER FROM $10 \times 4m$ ROOF INTO TANK $D = 1.5m$

$40 \times .065 \div (.75^2 \times \pi = 1.47\,m$ RISE

CYLINDER: $C = 16.7\,cm.$ $H = 23.4\,cm.$ CALCULATE $V.$

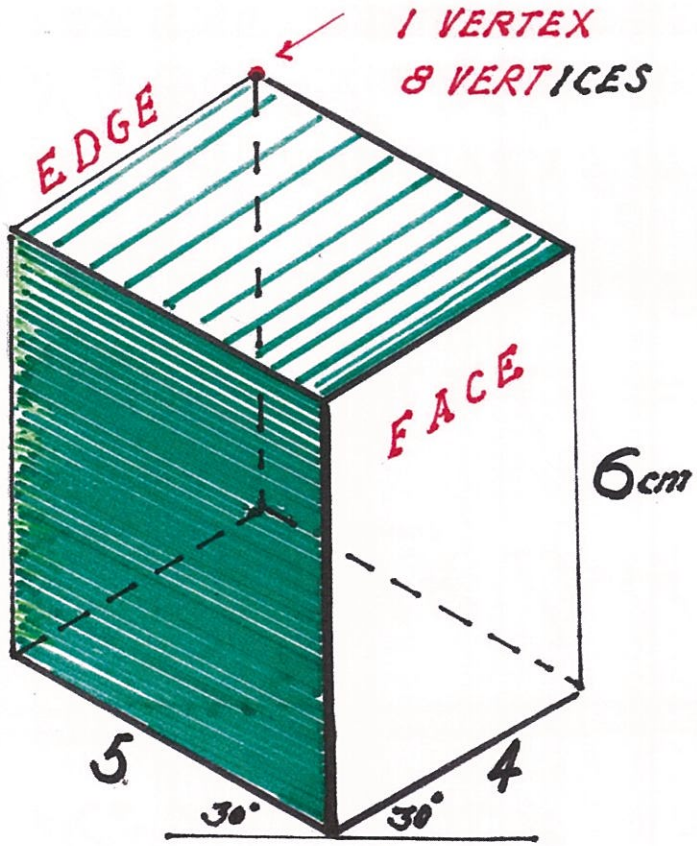
$(2R\pi = 16.7)$ D. $16.7 \div \pi \div 2 = \ldots (R)$ SQUARE IT $\ldots \times \pi \times 23.4 = 519.3\ cm^3$

168

RECTANGULAR PRISM

986.

691



VOLUME = 120 cm^3

5X4X6

SURFACE AREA

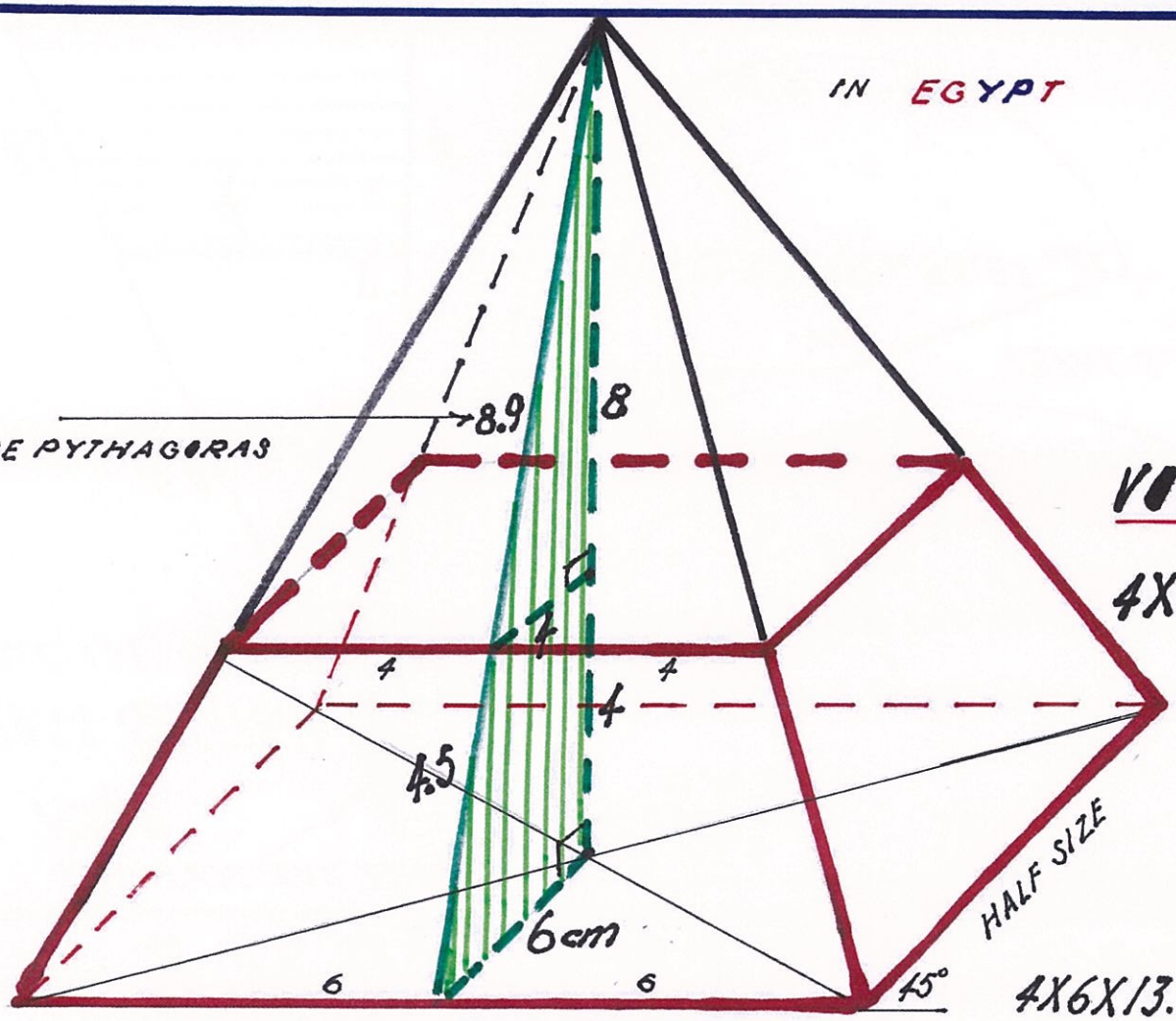
2X5X6 + 2X4X6 + 2X4X5 =

148 cm^2

TRUNCATED SQUARE PYRAMID

987.

AMIDST THE SAND IN EGYPT



VOLUME FRUSTRUM

$4X144-\frac{8}{3}X64=415\frac{1}{3}\ cm^3$

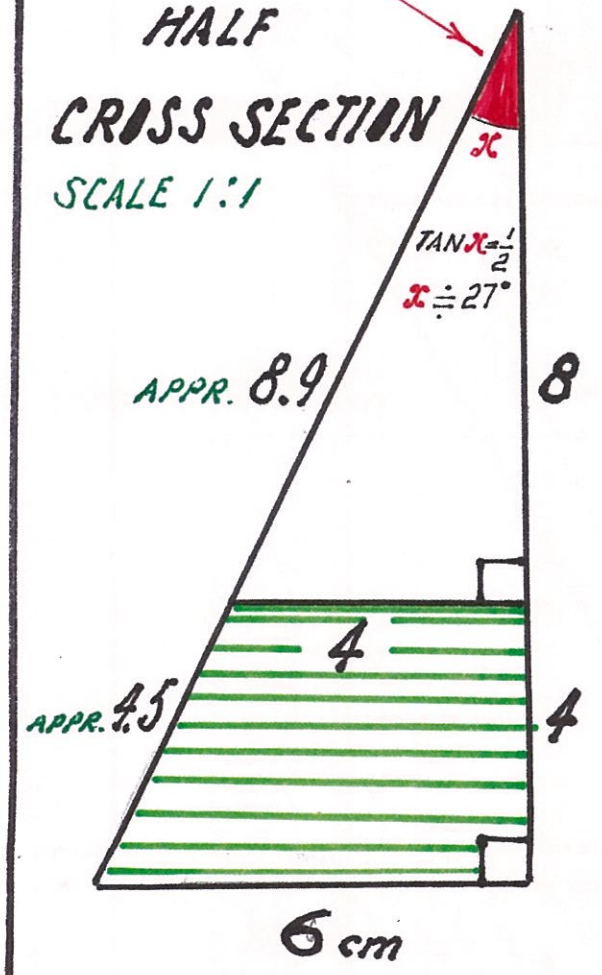
SURFACE AREA

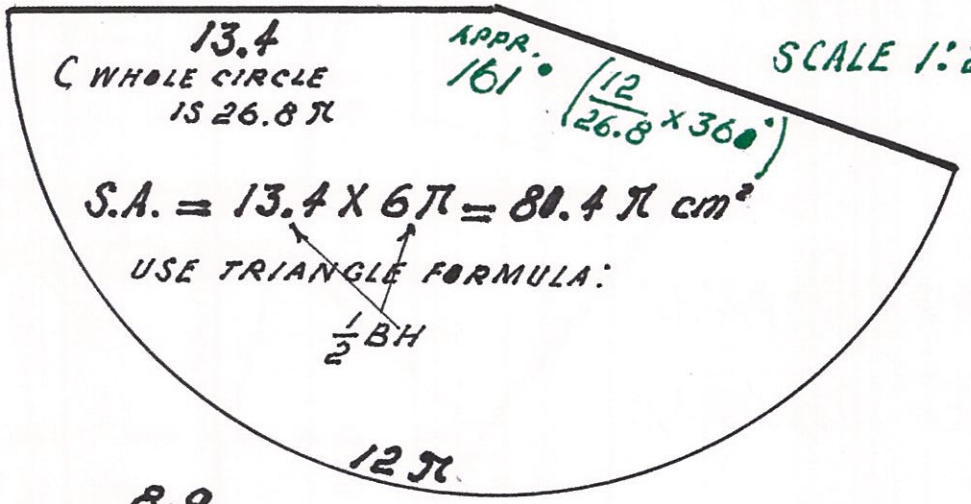
$4X6X13.4-4X4X8.9+64+144=387.2\ cm^2$

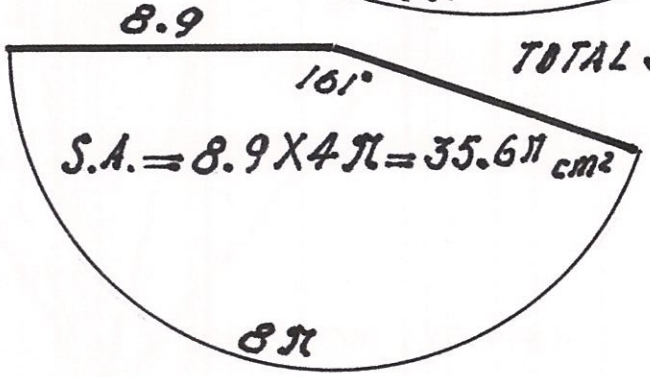
110

TRUNCATED CONE

988.







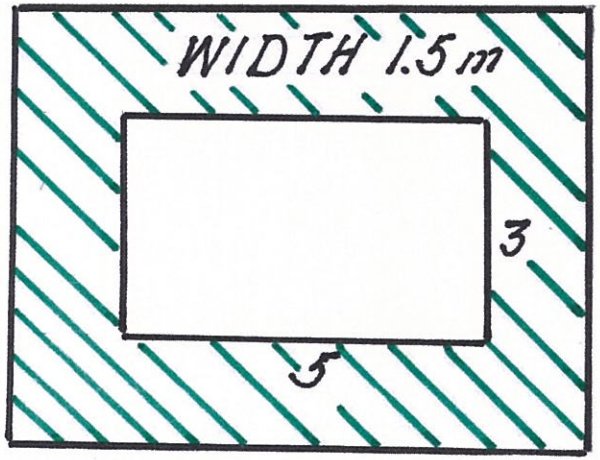
TOTAL S.A. = $44.8\pi + 36\pi + 16\pi \doteq$
$96.8\pi \doteq 304 \text{ cm}^2$

VOLUME FRUSTRUM $4 \times 36\pi - \frac{8}{3} \times 16\pi \doteq 318 \text{ cm}^3$

111

989

SCALE 1:100



GRAVEL PATH: DEPTH 6 cm

AREA $48-15 = 33 m^2$

VOLUME APPR. $2 m^3$ (33 X .06)

CYLINDER: $H = 21_{cm}$, $C = 44_{cm}$, $\pi = \frac{22}{7}$

$2R \times \frac{22}{7} = 44 \therefore R = 7_{cm}$

$V = 49 \times \frac{22}{7} \times 21 = 3168 cm^3$

$S.A. = 2 \times 49 \times \frac{22}{7} + 44 \times 21 = 1188 cm^2$

SOLIDS: TASK

164 SPHERE $R = 4$ cm SA & V

165 CONE $R = 7$cm, H 24 cm SA & V

166 SQUARE PYRAMID BASE 6 cm HEIGHT 4cm SA & V

167 CYLINDER $R = 6$ cm, $H = 8$ cm SA & V

168 SIMILAR PRISMS SA 1:3

WATER TANK: □ 4m, H 2m V IN m^3 & L

8 mm FROM 12X3 m INTO TANK, D 2m. RISE X

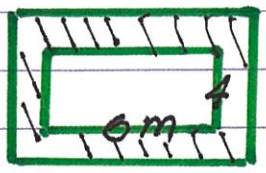
CYLINDER CIRCUMFERENCE 18 H 30 cm $V = x\ cm^3$

169 RECTANGULAR PRISM 5X6X7cm. SA & V

170 TRUNCATED PYRAMID FULL HEIGHT BASE 10 cm, H 12 cm HEIGHT FRUSTRUM 6 cm SA & V FRUSTRUM

171 TRUNCATED CONE R 5cm, H 12 cm H FRUSTRUM 6 cm SA & V

172

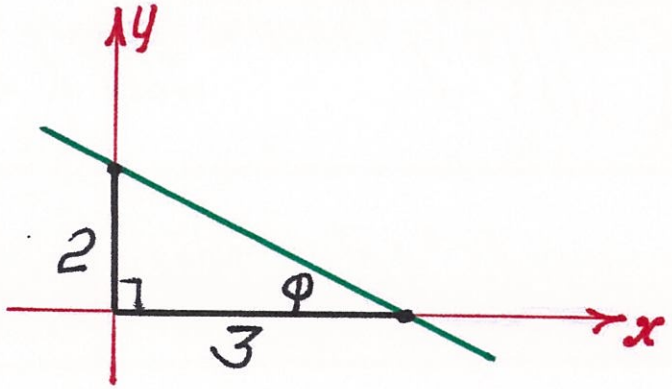


WIDTH 2m GRAVEL 4cm A & V

CIRCUMFERENCE 88 CYLINDER H 10 cm SA & V USE $\pi = \frac{22}{7}$

1038.

x-INTERCEPT $(3,0)$, y-INTERCEPT $(0,2)$



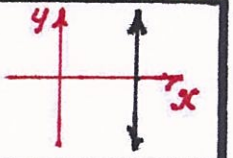
1. $\tan\varphi = \frac{2}{3}$

2. LINE LEANING LEFT

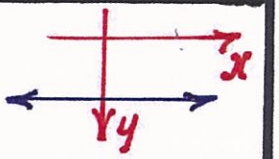
$\therefore$ m = NEG $\frac{2}{3}$

PARALLEL

LINE $\parallel$ y-AXIS MIDWAY $x=-1$ & $x=3$: $x = \frac{-1+3}{2} = 1$



LINE $\parallel$ x-AXIS EQUIDISTANT $y=1$ & $y=-3$: $y=-1$



EQUATION x-AXIS: $y=0$	EQUATION y-AXIS: $x=0$

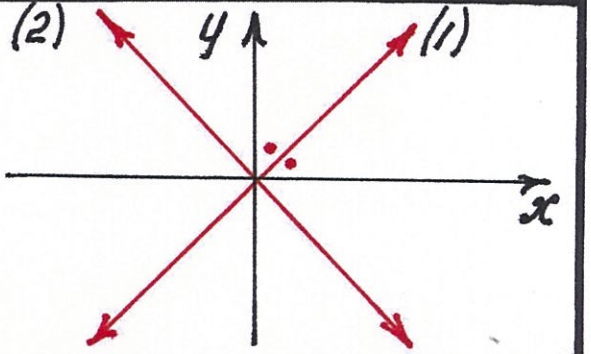
THE GRADIENT m & THE EQUATION 1041.

1. $m=3$, $b=-4$		$y=3x-4$
2. INCLINATION 64°, $b=5$ $m=\tan 64^\circ=2$		$y=2x+5$
3. x-INT. $(3,0)$, y-INT. $(0,1)$		$y=-\frac{1}{3}x+1$
4. LINE THROUGH $(1,4)\parallel y=2x+6$	$\therefore m=2$ $2+b=4$	$y=2x+2$
4. LINE THROUGH $(2,6)\perp y=-\frac{1}{3}x+1$	$\therefore m=3$ $6+b=6$	$y=3x$

LOCUS = PATH (LOCATION)

1041.

LOCUS (LOCI) OF P MOVING EQUIDISTANT FROM THE x AND y AXES

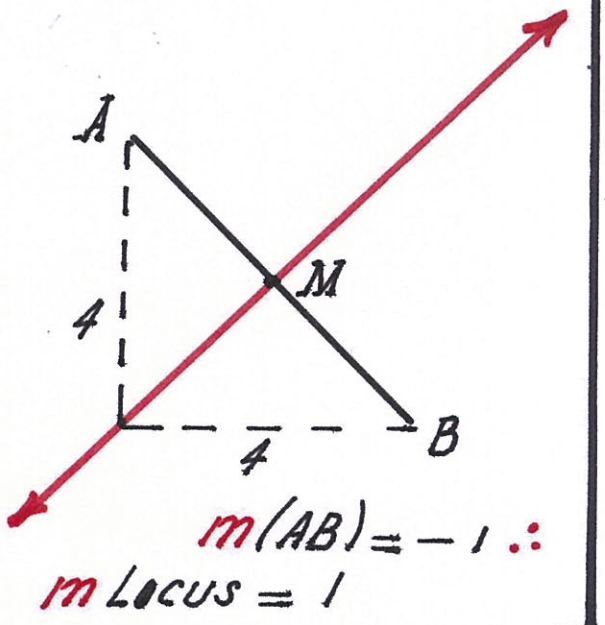


LOCUS OF P EQUIDISTANT FROM A(-1,6) & B(3,2).

IT'S THE PERPENDICULAR BISECTOR OF AB THROUGH THE MIDPOINT (1, 4)

$1+b=4$ EQUATION LOCUS: $y=x+3$

$-1+b=4$ EQUATION AB: $y=-x+5$



911

LINE THROUGH THE INTERSECTION OF | 1142.

$x+3y-4=0$ (1) & $3x-4y+1=0$ (2) AND $P(3,-1)$

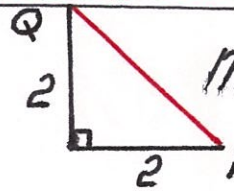
1. CALCULATING Q FIRST

USING SIMULTANEOUS EQUATIONS →

$3x+9y=12$ (1) ADJUSTED

$3x-4y=-1$ (2)

$13y=13,\ y=1,\ x=1,\ Q(1,1)$ $y_{SUB(1)}$



USING P: $m=-1,\ -3+b=-1$

EQ PQ: $\boxed{y=-x+2}$

2. WITHOUT CALCULATING Q FIRST

K-METHOD

P LIES ON PQ, SO $(3,-1)$ MUST SATISFY

$$(\overset{A}{x+3y-4})+K(\overset{B}{3x-4y+1})=0 \quad \text{SUB. } (3,-1)$$

$K=\frac{2}{7}$ SUB IN THE ABOVE & MULTIPLY BY 7 $\therefore$ $\boxed{y=-x+2}$

IF WRITTEN $KA+B=0$, K WOULD BE $\frac{7}{2}$ BUT GIVING ALSO

177

LINE PARALLEL TO $-x+2y+4=0$ AND PASSING THROUGH THE INTERSECTION OF $5x-2y+3=0$ & $2x+6y-7=0$

OR $y=\frac{1}{2}x-2$ $\therefore$ $m=\frac{1}{2}$

$(5x-2y+3)+K(2x+6y-7)=0$

$(5+2K)x+(6K-2)y+(3-7K)=0$

$\underline{+(6K-2)y}$ TO R.H.S.

$$m=\frac{5+2K}{2-6K}=\frac{1}{2} \quad \therefore 10+4K=2-6K \therefore K=-\frac{4}{5}$$

SUB. $\therefore$

BEWARE

CROSS MULTIPLY

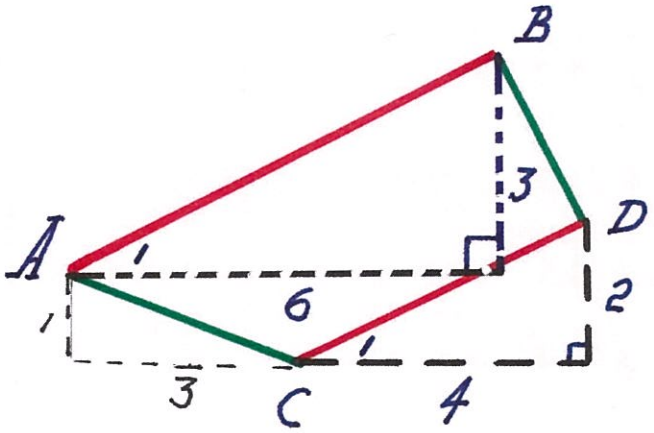
$17x-34y+43=0$

A(1,2), B(6,5), C(3,1), D(7,3)

1044.

PROVE ABCD IS A TRAPEZIUM e.g 2 OPPOSITE PARALLELS

PROCEDURE: 1. PLOT THE POINTS. 2. DRAW THE QUADRILATERAL.
3. CALCULATE THE GRADIENTS OF THE SIDES THAT LOOK PARALLEL BY COMPLETING THE RIGHT TRIANGLES. **DO NOT DRAW THE AXES;** THEY ONLY OBSTRUCT THE PICTURE



$$m_{AB} = \tan \hat{A}_1 = \frac{1}{2}$$
$$m_{CD} = \tan \hat{C}_1 = \frac{1}{2}$$
$$\therefore AB \parallel CD$$

q.e.d
quod erat demonstrandum
THAT'S WHAT WE HAD TO PROVE

611

DISTANCE P(-1,6) TO $4x-3y=-2$ | 1045.

PROCEDURE:

1. CONVERT EQUATION TO GENERAL BUT IGNORE $=0$

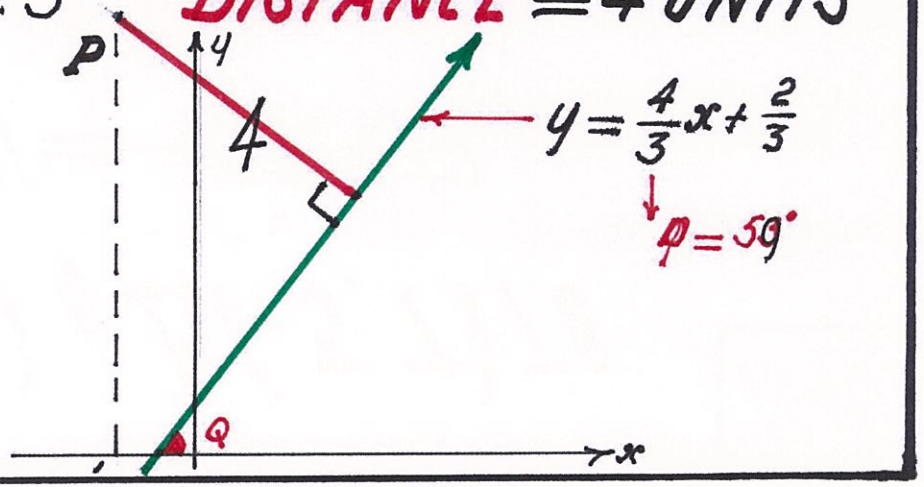
$$\underset{Ax}{4x}\underset{+By}{-3y}\underset{+C}{+2} \quad \begin{matrix} A=4 \\ B=-3 \end{matrix}$$

IGNORE (ABSOLUTE VALUE)

2. SUB. $x=-1$ & $y=6$ TO GIVE -20

3. DIVIDE BY $\sqrt{A^2+B^2}=\sqrt{16+9}=5$

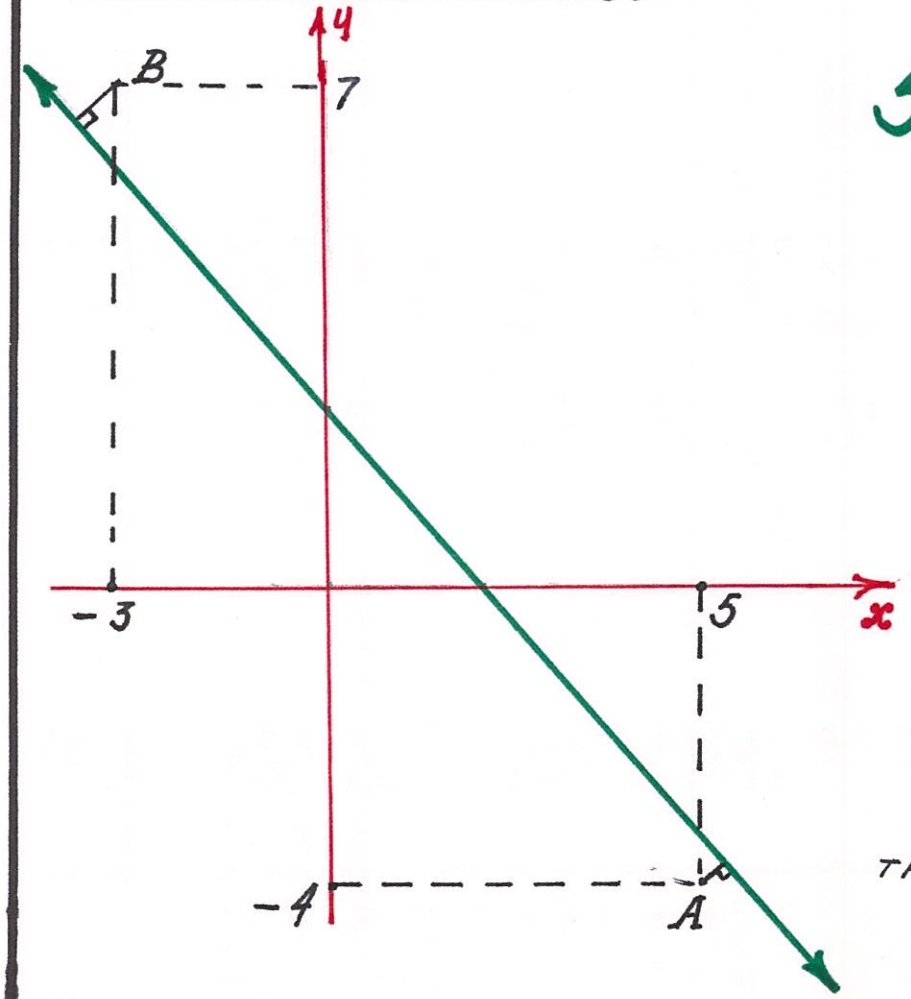
DISTANCE $=4$ UNITS



081

1046.

DISTANCE A(5,-4) & B(-3, 7) TO



$5y = -6x + 12$ CONVERT TO

$-6x - 5y + 12$

SUB. A $= 2$ CLOSER

SUB. B $= -5$

↑ SHOWS DIFFERENT SIDE OF THE LINE

THERE'S NO NEED TO DIVIDE BY THE COMMON DENOMINATOR

$\sqrt{A^2 \quad B^2}$
$\sqrt{36 + 25}$

LINE THROUGH $A(-4,-3)$

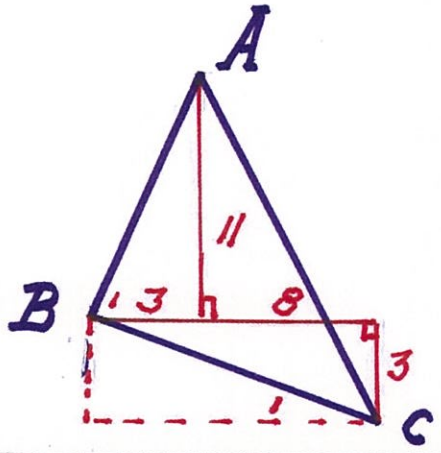
1047.

$m=\frac{3}{2}$ ALWAYS THINK $mx+b=y$

$-6+b=-3 \therefore y=\frac{3}{2}x+3$ OR

$2y=3x+6$

TRIANGLE $A(1,4), B(-2,-7), C(9,-11)$



$m_{AB}=\frac{11}{3}$, $m_{BC}=-\frac{3}{11}$ $\Big\}_{A\hat{B}C=90}$

TAN $\hat{B}_1$ TAN $\hat{C}_1$ TAN

NO SIGN = + = LEANING RIGHT SEE

SEE LEANING LEFT

STRAIGT LINES: TASK 1.

174 X-INT. (4.0), Y-INT. (0.3) | PARALLEL LINE y-AXIS | MIDWAY -2 & 5

LINE // X-AXIS EQUIDISTANT $y=3$ & $y=-5$ | EQUATIONS X-AXIS, Y-AXIS

175 $m=4$, $b=-7$ | INCLINATION $50°$, $b=3$ | X-INT. (5,0), Y-INT. (0,2)

LINE THROUGH (2,5) $\parallel y=3x+1$ | LINE THROUGH (3,8) $\perp y=-\frac{1}{3}+2$

176 LOCUS OF P(PATH) EQUIDISTANT FROM A(-1,-1) & B(3,7)

177 LINE THROUGH INTERSECTION OF $3x+y-4=0$ (1) & $4x-3y-14=0$ (2) AND P(-4,1)

178 LINE // $-2x+y+5=0$ AND PASSING THROUGH THE INTERSECTION OF $3x-3y-6=0$ (1) & $3x+2y-4=0$ (2)

179 A(0,4), B(8,6)

C(3,1), D(7,2)

PROVE AB // CD

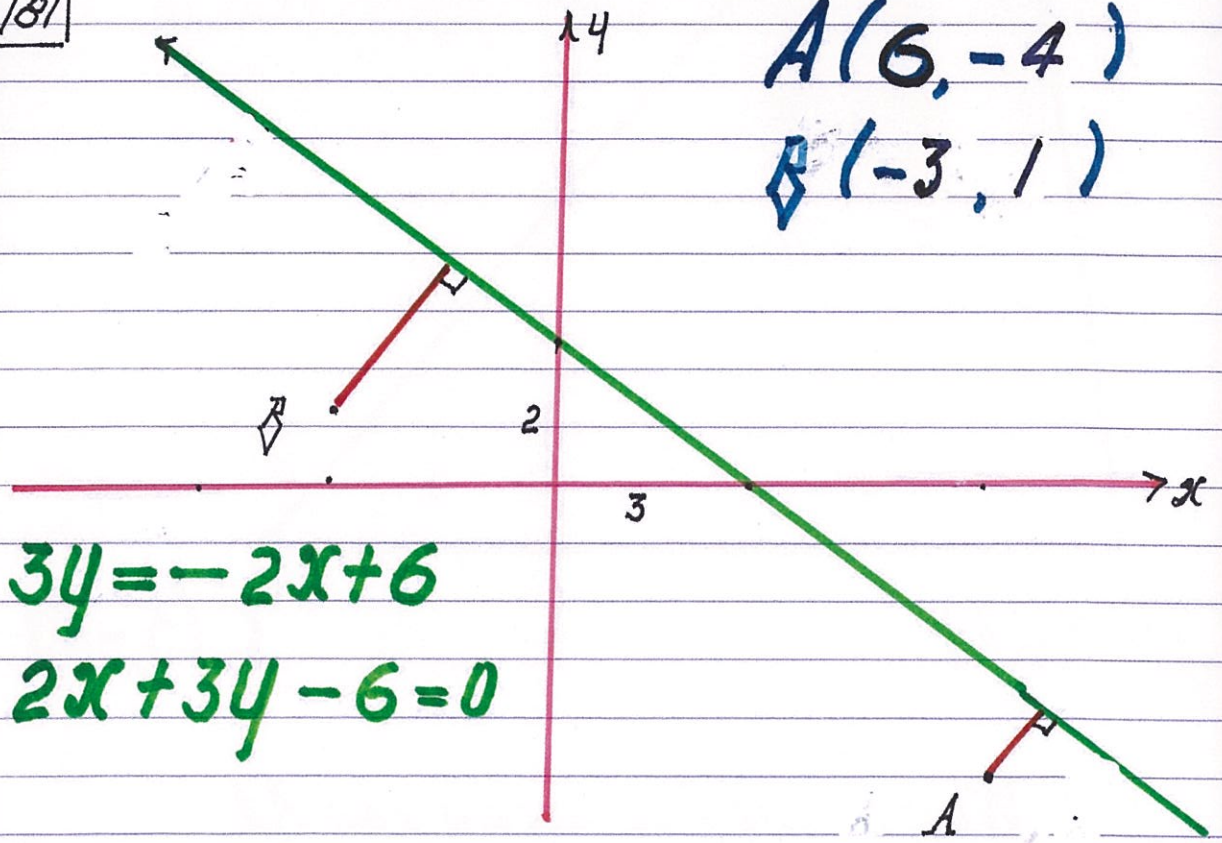
TASK 2

180

DISTANCE

$P(-2,8)$ to $2x-7y=14$

181



$A(6,-4)$

$B(-3,1)$

$3y=-2x+6$

$2x+3y-6=0$

182

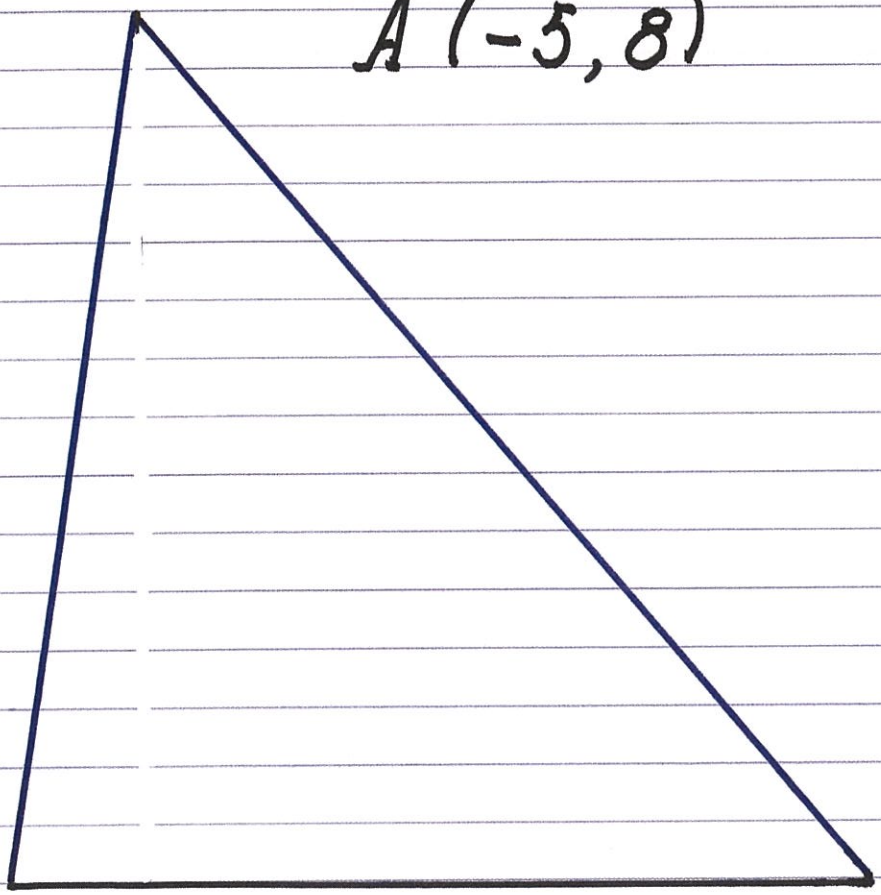
LINE THROUGH

$A(-6,-2)$

$m=2$

TASK 3

183



WRITE THE EQUATION FOR $\overleftrightarrow{BA}$

CHECK IF A IS ON IT.

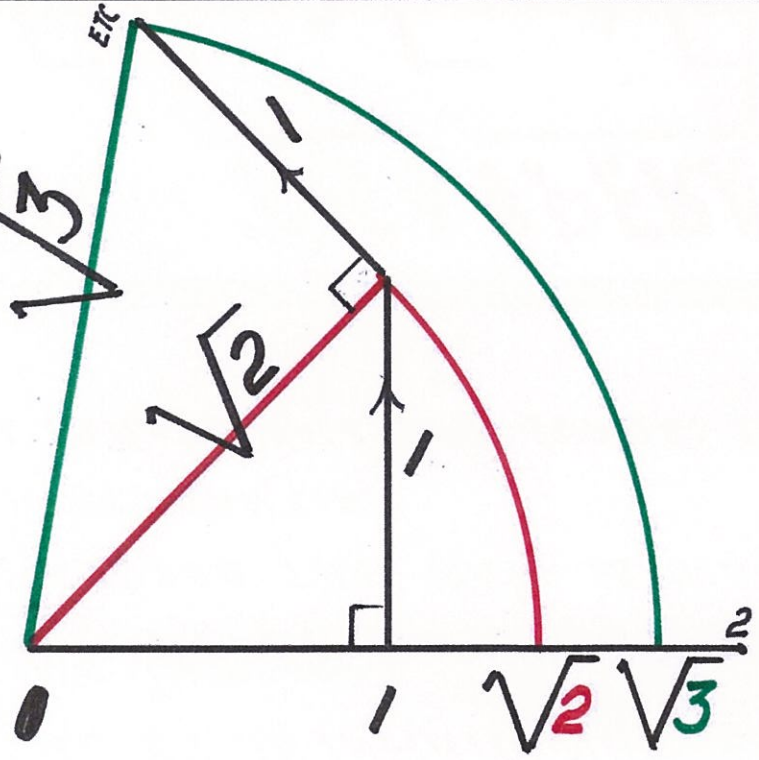
WRITE THE EQUATION FOR CA

CHECK IF A IS ON IT.

AB SURDS IRRATIONALS

1056.

AS ACCURATE AS RATIONALS ON THE NUMBERLINE BUT NEITHER RECURRING NOR TERMINATING BY CALCULATOR.



– PYTHAGORAS –

$$\sqrt{3} = 1.732050808\ldots$$

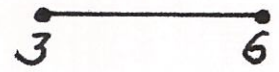
$$\sqrt{2} = 1.414213562\ldots$$

NOTE: WHEN SQUARED ··· 2

BECAUSE $(\sqrt{2})^2 = 2$

$(1.414213562)^2 = 1.\dot{9}$

(n) IRRATIONALS > (n) RATIONALS BECAUSE EACH RATIONAL CAN BE MULTIPLIED BY AN INFINITE NUMBER OF IRRATIONALS TO FORM NEW ONES. THEREFORE, $3 \leqslant x \leqslant 6$ $(x \in Q)$ GRAPHED AS



-AND ACCEPTED THAT WAY- IS INACCURATE BECAUSE THERE SHOULD BE MORE HOLES THAN LINE.

NOT ALL IRRATIONALS ARE SURDS: TRANSCENDENTALS (UNIQUE)

e (2.718...), π (3.14...)

THE 4 OPERATIONS

$2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$

$5\sqrt{6} - \sqrt{6} = 4\sqrt{6}$

$3\sqrt{2} \times 2\sqrt{3} = 6\sqrt{6}$

$8\sqrt{15} \div 4\sqrt{5} = 2\sqrt{3}$

USE 1.414

THE RECIPROCAL OF $\sqrt{2}$ IS

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = .707$$

$$(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})$$

CONJUGATE (LINKED) SURDS: USE THE DIFF. OF 2 SQUARES

$$5-3=2$$

1059.

$$t = 2\sqrt{3} \therefore t^4 - 2t^2 + 1 =$$

$$16 \times 9 - 2 \times 4 \times 3 + 1 = 121$$

(144 - 24 + 1)

SEE ORDER OF OPERATION — FIRST — THEN +1

681

DIRECT ANSWERS!

$$\sqrt{4} \times \sqrt{4} \quad \text{OR} \quad (\sqrt{4})^2 = 4$$

$$\sqrt{5} \times \sqrt{5} \quad \text{OR} \quad (\sqrt{5})^2 = 5$$

ETC...

061

1061.

$\sqrt{16x^{16}}$ OR $(16x^{16})^{\frac{1}{2}} = 4x^8$ A ROUTINE

$(3-\sqrt{5})(3+\sqrt{5}) =$ THINK ONLY 9-5; WRITE 4

BETWEEN WHICH TWO WHOLE NUMBERS INTEGERS (INTEGRITY) LIES $\sqrt{13}$? (THINK $\sqrt{9}$ & $\sqrt{16}$) 3 & 4

SIMPLIFYING SURDS: LOOK FOR SQUARES

$\sqrt{125} - \sqrt{5} - \sqrt{20} =$ THINK ONLY $5\sqrt{5} - \sqrt{5} - 2\sqrt{5} = 2\sqrt{5}$

CLUE!

1161.

$$(\sqrt{5}-\sqrt{3})^2+\sqrt{60}$$

PERFECT SQ. ROUTINE

$$5-2\sqrt{15}+3+2\sqrt{15}=8$$

SQ. FIRST, DOUBLE PRODUCT, SQ. LAST

$$\sqrt{32}+\sqrt{98}-\sqrt{50}=$$ THINK $4\sqrt{2}+7\sqrt{2}-5\sqrt{2}=$ WRITE $6\sqrt{2}$

$$\sqrt{18}+\sqrt{8}=5\sqrt{2}=\sqrt{50}=\sqrt{a}$$ $\sqrt{a}=$ $\therefore a=50$

SURDS: TASK

187

$3\sqrt{5}+2\sqrt{5}$

$3\sqrt{7}-\sqrt{7}$

$3\sqrt{2}\times 2\sqrt{3}$

$7\sqrt{21}\div\sqrt{3}$

188

RECIPROCAL OF $\sqrt{3}$

$(\sqrt{7}-\sqrt{3})(\sqrt{7}+\sqrt{3})$

189

$t=3\sqrt{2}$

$t^4-3t^2+2=\cdots$

$\sqrt{3}\times\sqrt{3}$

$\sqrt{7}\times\sqrt{7}$

190

$\sqrt{25a^6}$

$(\sqrt{5}-2)(\sqrt{5}+2)$

$\sqrt{115}+\sqrt{63}-\sqrt{7}$

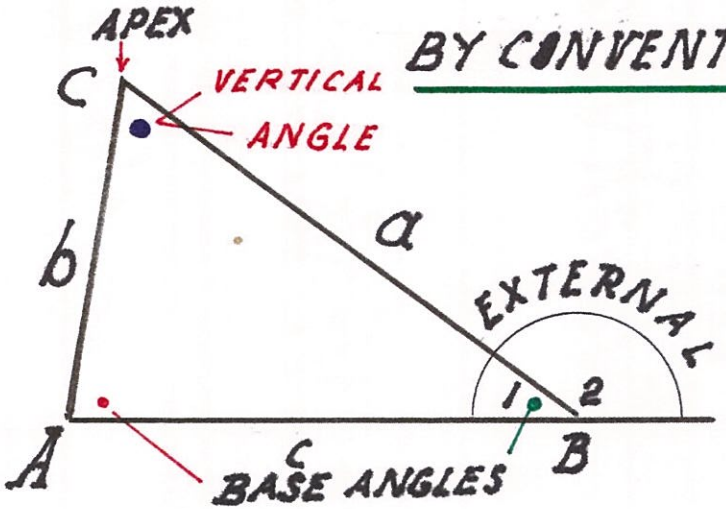
$(\sqrt{7}-\sqrt{5})^2+2\sqrt{35}$ 191

$\sqrt{50}+\sqrt{2}-\sqrt{18}$

$\sqrt{a}=\sqrt{32}+\sqrt{50}$

TRIANGLES

1076.



BY CONVENTION : a OPPOSITE $\hat{A}$, ETC. ...

$\hat{A} + \hat{B}_1 + \hat{C} = 180^\circ$ SEE ANGLES

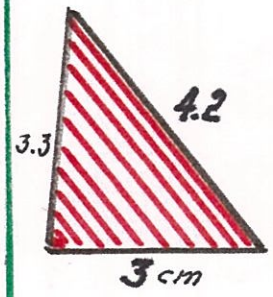
$\hat{B}_1 + \hat{B}_2 = 180^\circ \therefore \hat{B}_2 = \hat{A} + \hat{C}$

EXT. ANGLE = SUM REMOTE ANGLES

ACCORDING TO SIDES

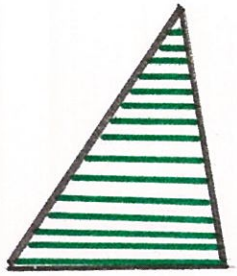
EQUILATERAL

ISOSCELES

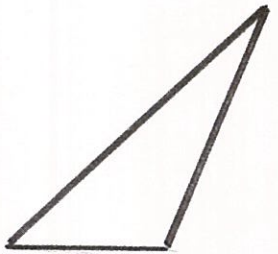


SCALENE

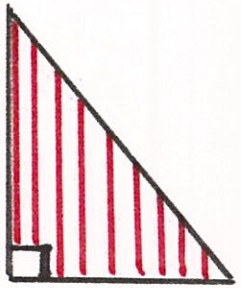
ACCORDING TO ANGLES



ACUTE



OBTUSE



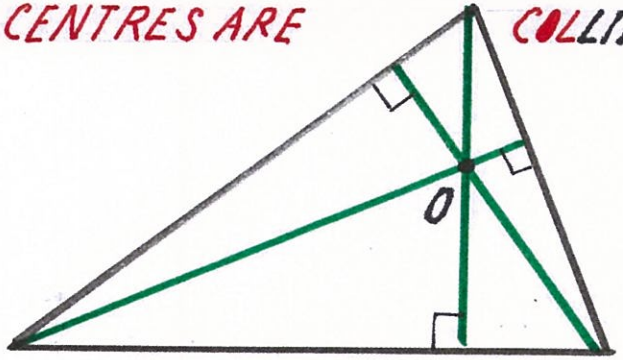
RIGHT

SPECIAL CIN CURRENT LINES

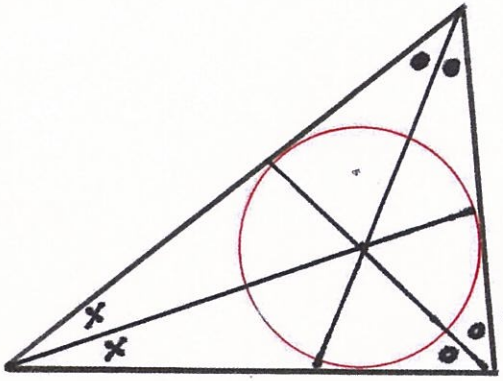
1077

161

THE 4 CENTRES ARE COLLINEAR!

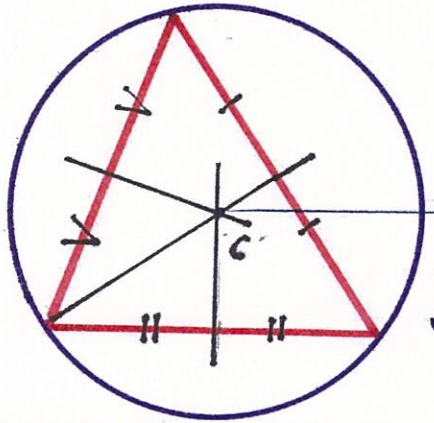


ALTITUDES. O = ORTHO CENTRE



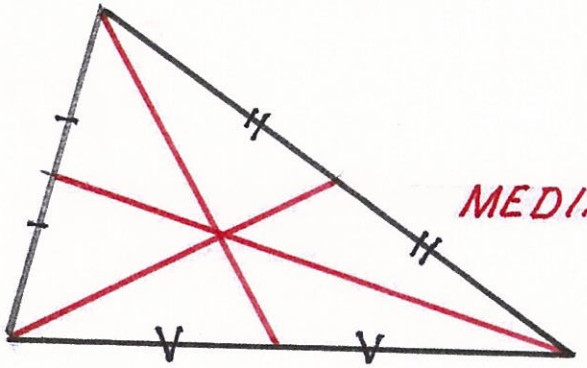
INCENTRE
INSCRIBED CIRCLE

ANGLE BISECTORS



CIRCUM-CENTRE
CIRCUM-SCRIBED CIRCLE

BISECTORS OF SIDES

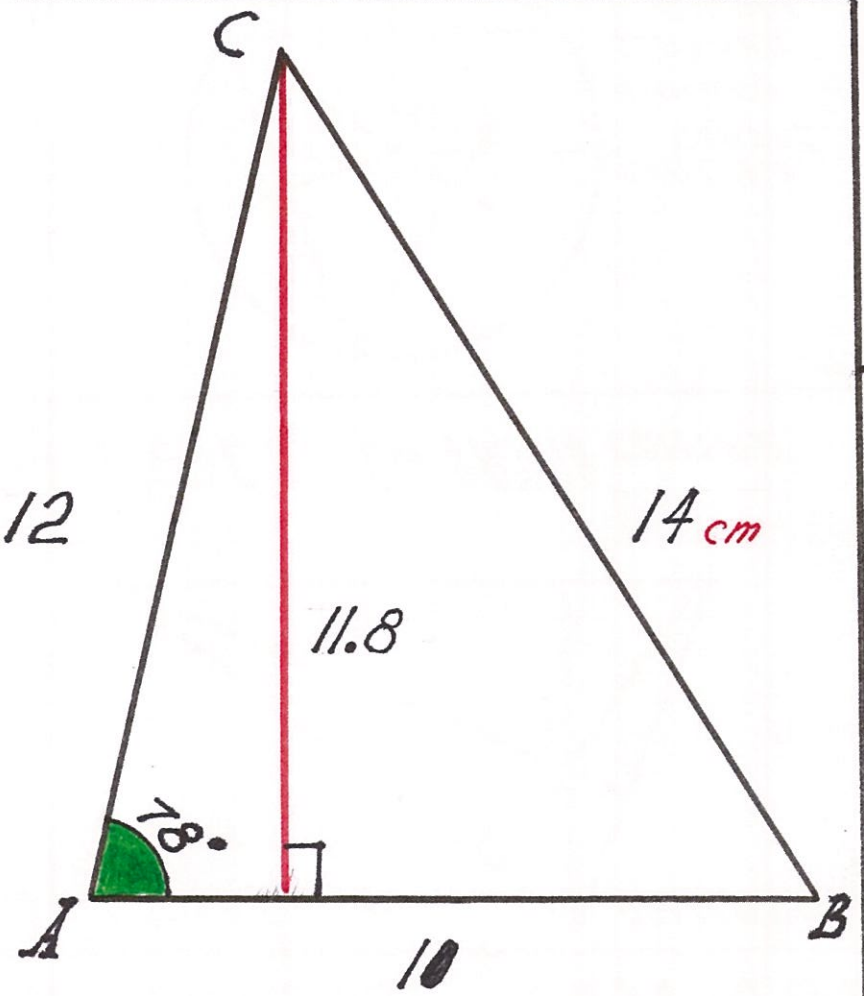


MEDIANS

CENTROID (BALANCE CENTRE)

AREA TRIANGLE DEPENDING ON DATA.

1078.



1. THINK ONLY: $\frac{1}{2}$ B.H.

WRITE

DO $5 \times 11.8 = 59$ cm²

2. THINK ONLY: $\frac{1}{2} \times 10 \times 12 \sin 78$

SAME FORMAT

DO $60 \times \sin 78 = 58.7$ cm²

3. THINK ONLY: $\sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{1}{2}(a+b+c)$

AGAIN!

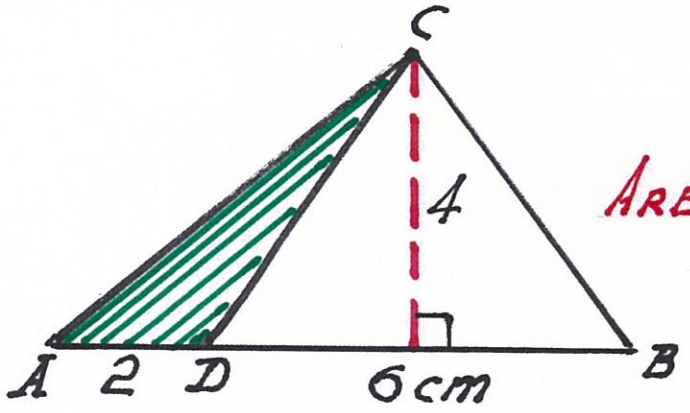
DO $\sqrt{18 \times 8 \times 6 \times 4} = 58.8$ cm²

PRESS 72 X 48 OF COURSE

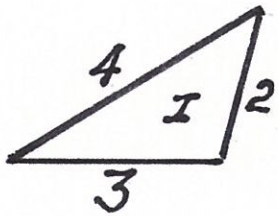
195

961

1079.

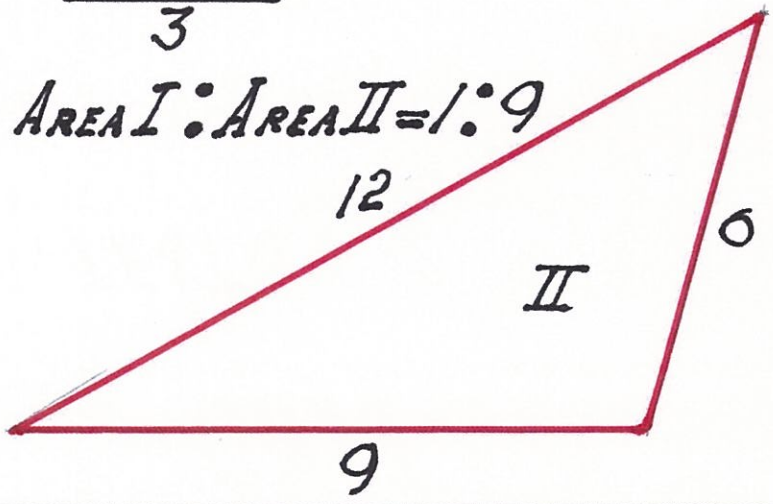


AREA $\triangle ACD : \triangle BCD = 4 : 12 = 1 : 3$



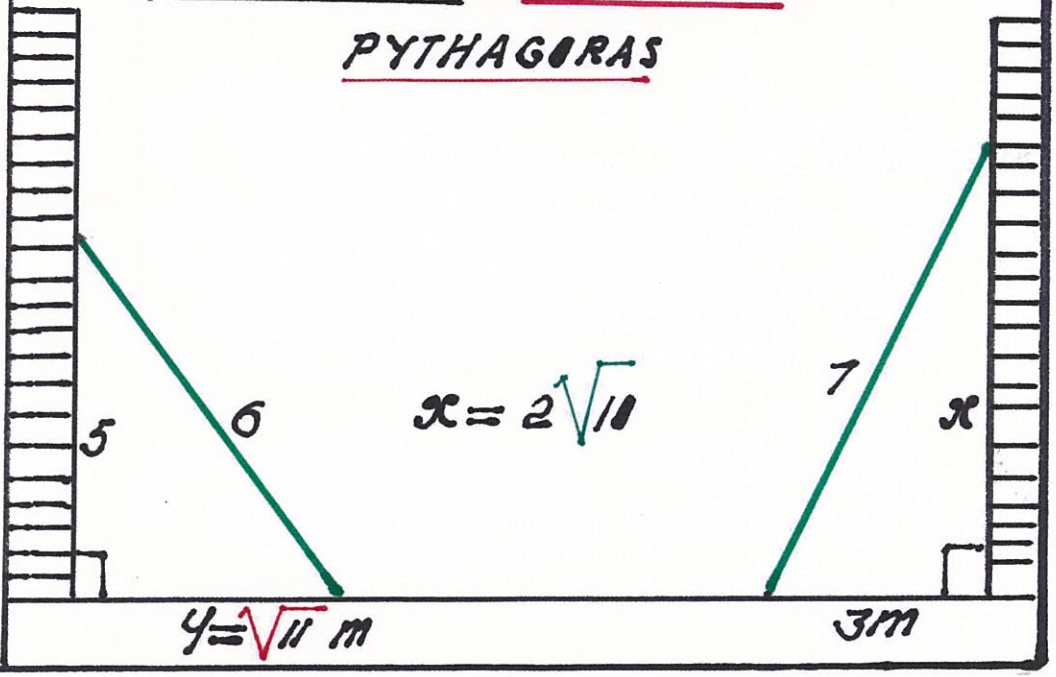
SIDES 1 : 3

AREA I : AREA II = 1 : 9



LADDER QUESTIONS

PYTHAGORAS

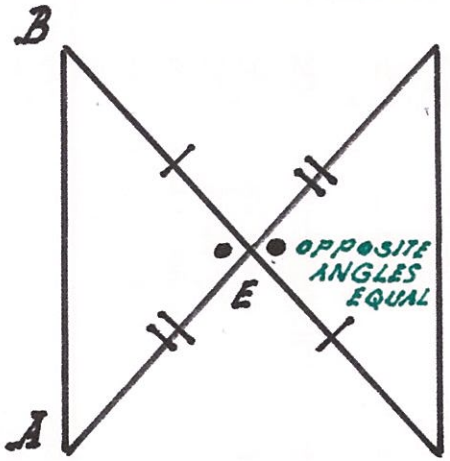


SAME SIZE ≡ CONGRUENT TRIANGLES

1080.

4 TESTS: IF 3 SPECIFIC PARTS ARE EQUAL, THE 3 OTHERS ARE

S.A.S



SIDE. ANGLE. SIDE

RIGHT ORDER!

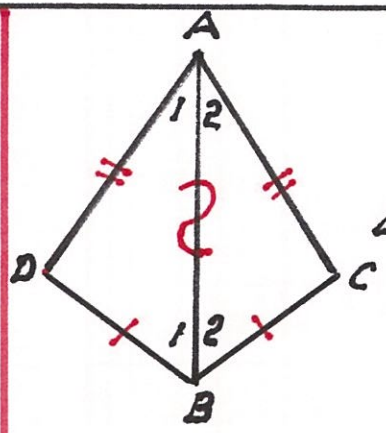
$\triangle ABE \equiv \triangle CDE$ ∴

CONGRUENT TO

$\hat{A} = \hat{C}$

OPPOSITE ONE STROKE

S.S.S

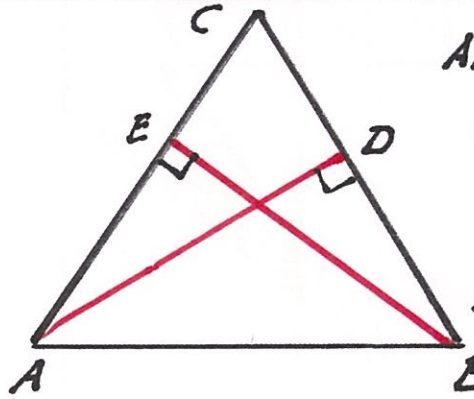


$\triangle ABD \cong \triangle ABC$ ∴

$\hat{A}_1 = \hat{A}_2$

$\hat{B}_1 = \hat{B}_2$

$\hat{C} = \hat{D}$



$AD = BE$ (GIVEN)

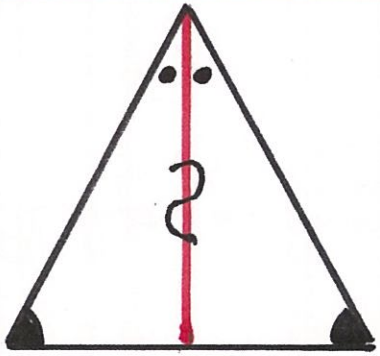
$\hat{E} = \hat{D} = 90°$

∴

$\triangle ABE \equiv \triangle BAD$

R.H.S

RIGHTANGLE, HYPOTENUSE, SIDE.



A.A.S

SIMILAR TRIANGLES

SAME SHAPE DIFFERENT SIZE

1081.

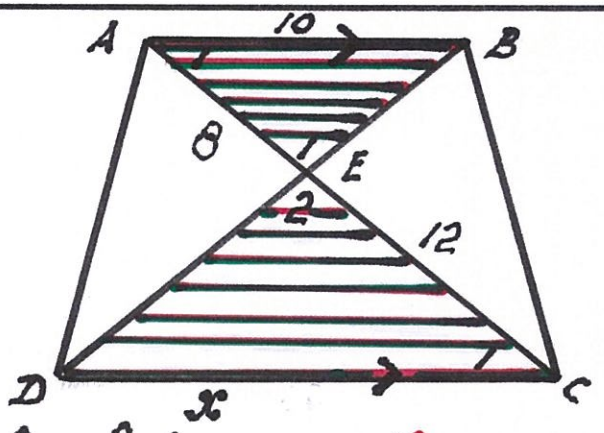
1. IF 2 ANGLES (NOT ANGELS!) ARE EQUAL, THE 3RD ONE IS (ANGLE SUM 180°)

2. ONLY SIDES OPPOSITE EQUAL ANGLES ARE IN PROPORTION

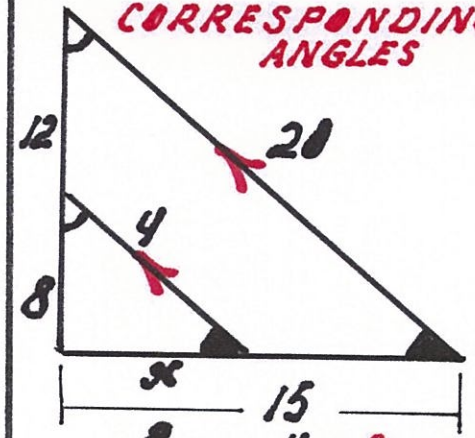
961

RATIO

EDGES SIMILAR SOLIDS 2:5 | $S.A._1 : S.A._2 = 4:25$ | $V_1 : V_2 = 8:125$

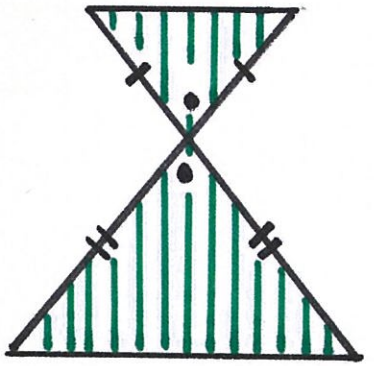


$\hat{A}_1 = \hat{C}_1$ (ALTERNATE)
$\hat{E}_1 = E_2$ (OPPOSITE)
$\triangle ABE \;|||\; \triangle CDE$
$8:12 = 10:x \;(15)$



$\frac{8}{20} = \frac{y}{20} = 8$

$\frac{8}{20} = \frac{x}{15} = 6$

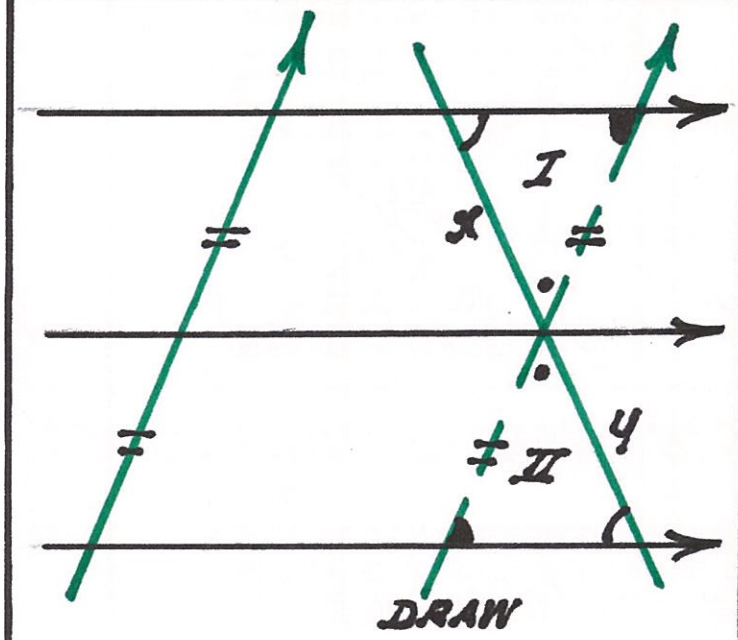


ISOSCELES TRIANGLES WITH EQUAL VERTICAL ANGLES ARE SIMILAR.

INTERCEPT PROPERTIES ON TRANSVERSALS (ACROSS / RUN) 1082.

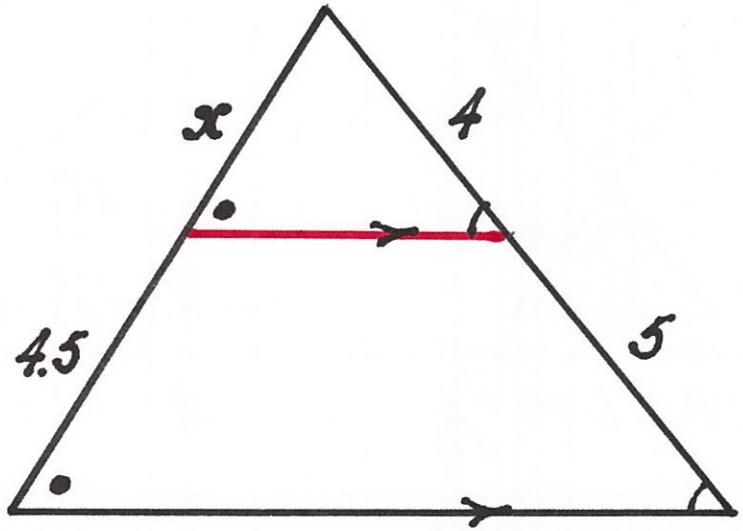
BY A FAMILY OF PARALLEL LINES

661



$\triangle I \sim \triangle II$ ∴

$x = y$ (OPPOSITE ∠'s)



EQUAL RATIOS

$$\frac{4}{5} = \frac{x}{4.5} = 3\frac{3}{5}$$

TIMES (shortcut)

DIVIDE

SIMILAR TRIANGLES

$$\frac{4}{9} = \frac{x}{x+4.5}$$

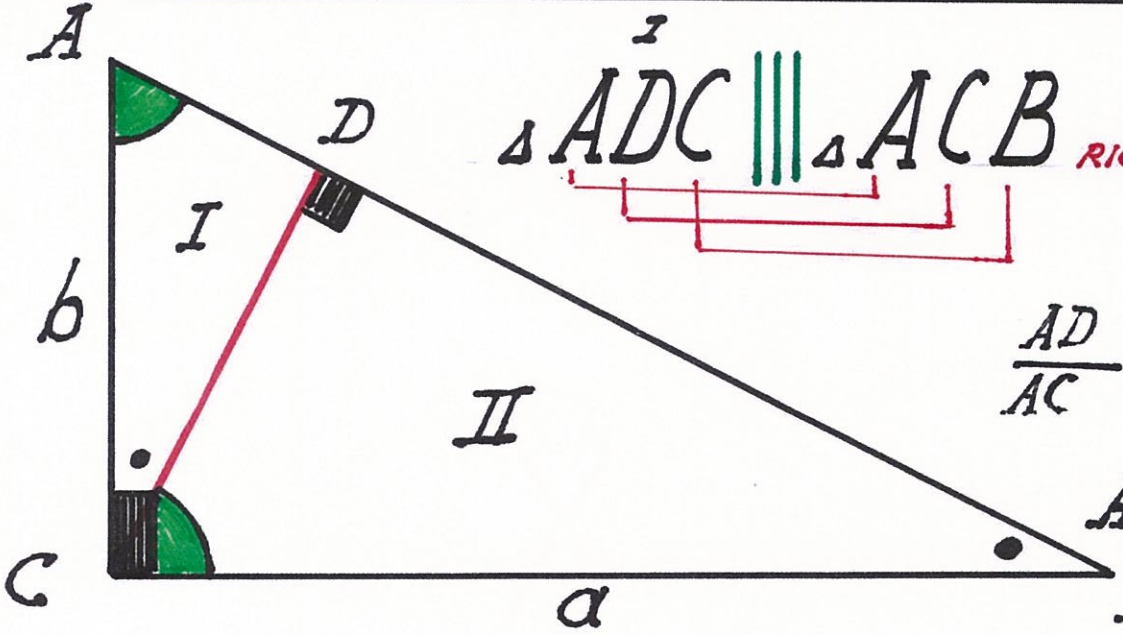
2 1

CROSS MULTIPLY

$x = 3\frac{3}{5}$

PYTHAGORAS PROOF

1183



I $\triangle ADC \equiv \triangle ACB$ RIGHT ORDER! $\therefore$

$\frac{AD}{AC} = \frac{AC}{AB}$ $\therefore$

$AC^2 = AB.AD$ (1)

II $\triangle BCD \equiv \triangle BAC \therefore \frac{BC}{AB} = \frac{BD}{BC}$ $\therefore$ $BC^2 = AB.BD$ (2)

$BC^2 + AC^2 = AB(AD+BD) = AB^2$ OR $\boxed{a^2+b^2=c^2}$

200

TRIANGLES: TASK

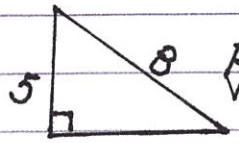
194

COPY THE CONSTRUCTIONS

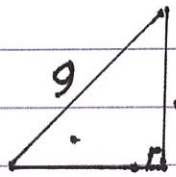
CONSTRUCT △ABC: $\overline{AB}=5$cm, $\overline{AC}=$, $\overline{BC}=$
MEASURE: H & $\hat{A}$. FIND P & A (3 WAYS)

196 $\overline{AD}=4$cm $\overline{OB}=8$cm $H=6$cm

TRIANGLE SIDES
2, 4, 6 & 8, 16, 24



BASE x

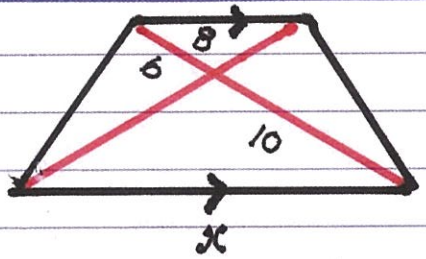


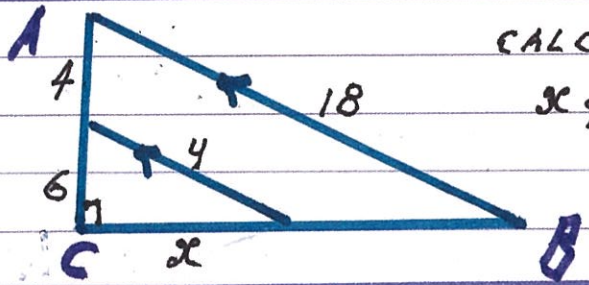
BASE 5

197

REMEMBER THE 4. THREE LETTER-TESTS

198



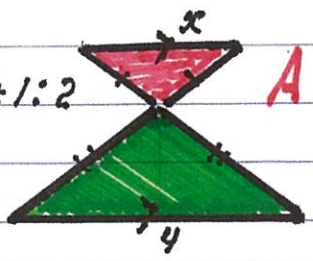


CALCULATE $\overline{BC}$
x & y

EDGES SIMILAR SOLIDS

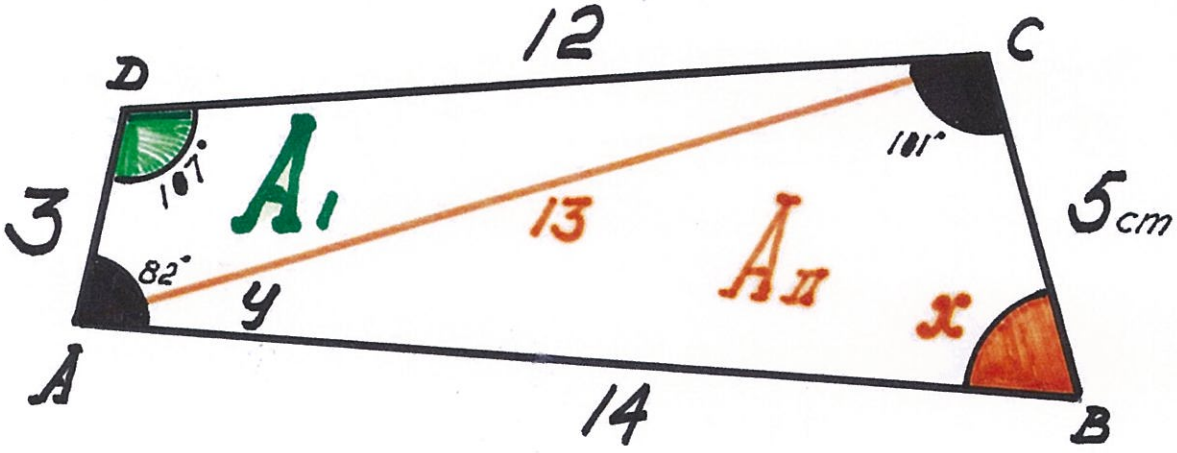
3:7

x:y = 1:2



A : A =

914.



DIRECT BY CALCULATOR: $x = 361 - 82 - 117 - 111 = 71°$

THINK $\frac{1}{2}$X12X3

$A_1 = 18 \underset{D°}{} \text{SIN } 117 = 17.2 \text{ cm}^2$

$A_{II} = 35 \text{ SIN } 71 \doteq 32.9 \text{ cm}^2$

LEAVE FULL ANSWER

THINK $\frac{1}{2}$X14X13

$91 \underset{D°}{} \text{SIN } y = 32.9$

DIVIDE BY

$\text{SIN } y = 32.9 \div 91 = \ldots$ INV. SIN $21° = y$

RECIPROCALS

1091.

CO-SECANT (COSEC) $= \frac{1}{SIN}$

COSEC 50 : DO SIN 50 ¢ 1/x

SECANT (SEC) $= \frac{1}{COS}$

SEC 50 : DO COS 50 ¢ 1/x BUTTON

CO-TAN (COT) $= \frac{1}{TAN}$

COT 50 : DO TAN 50 ¢ 1/x OR

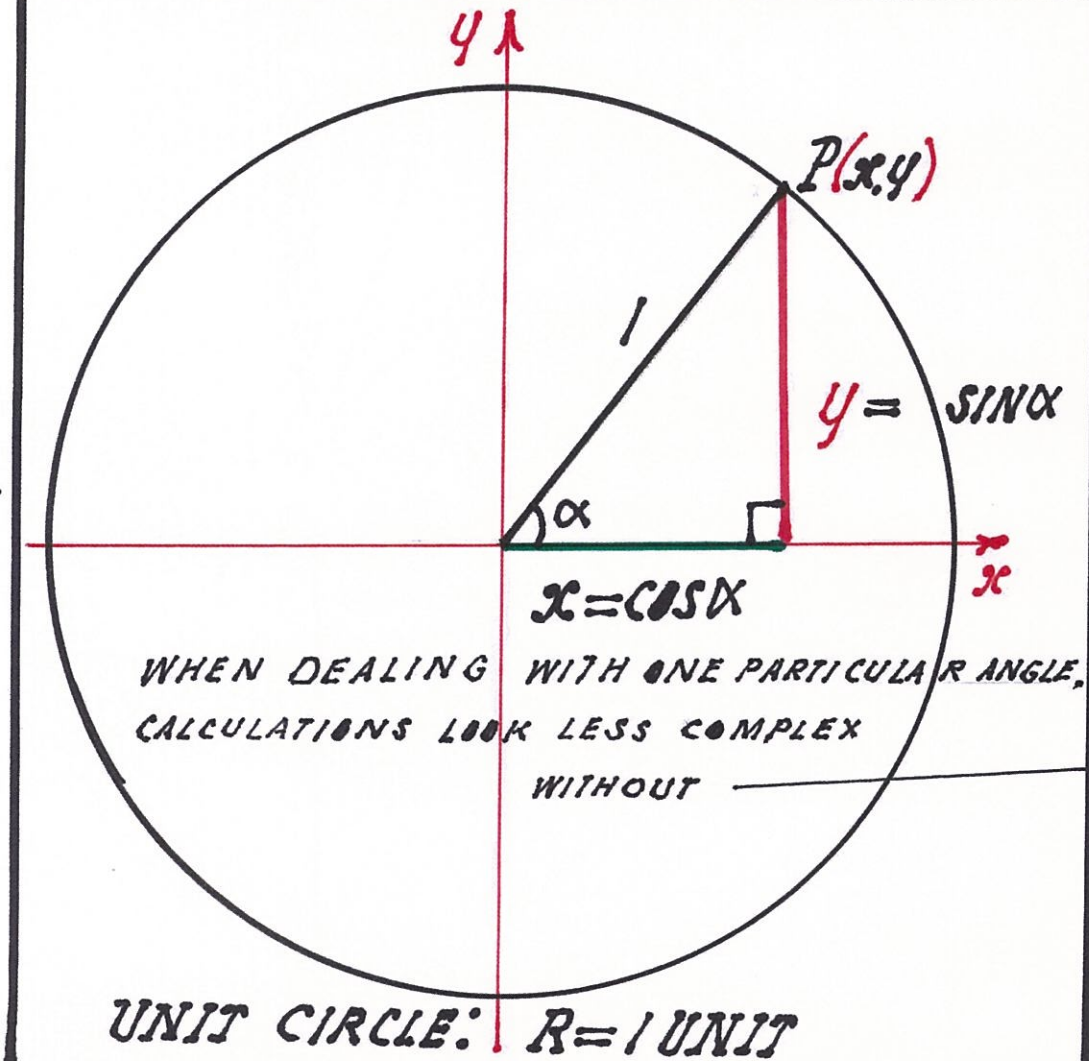
RECIPROCAL AND

DO TAN 40

COMPLEMENT OF TAN : COT 30 = TAN 60

4 IDENTITIES

1092.



WHEN DEALING WITH ONE PARTICULAR ANGLE, CALCULATIONS LOOK LESS COMPLEX WITHOUT

UNIT CIRCLE: R = 1 UNIT

1. $TAN = \frac{SIN}{COS}$

2. $SIN^2 + COS^2 = 1$

DIVIDE BY COS^2

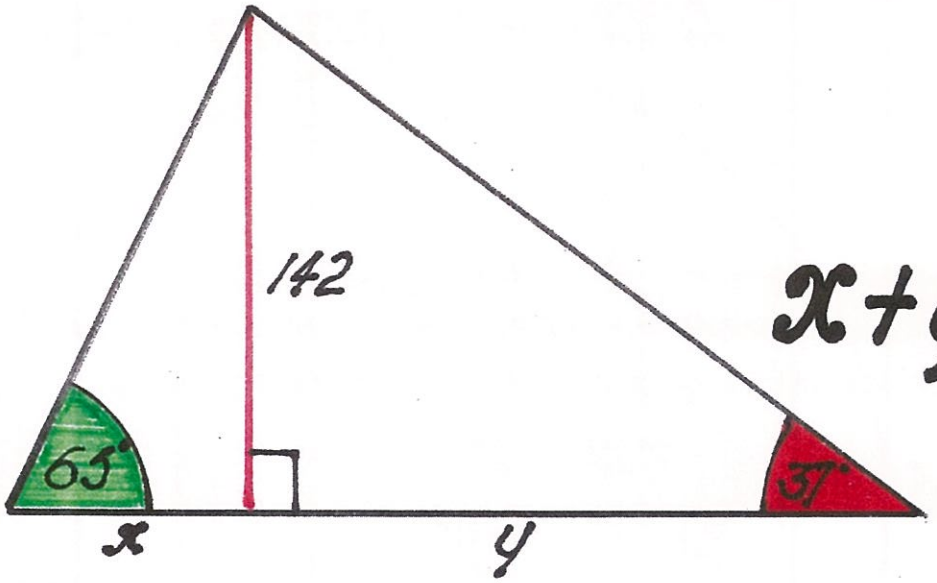
3. $TAN^2 + 1 = SEC^2$

DIVIDE BY SIN^2

4. $1 + COT^2 = COSEC^2$

1092

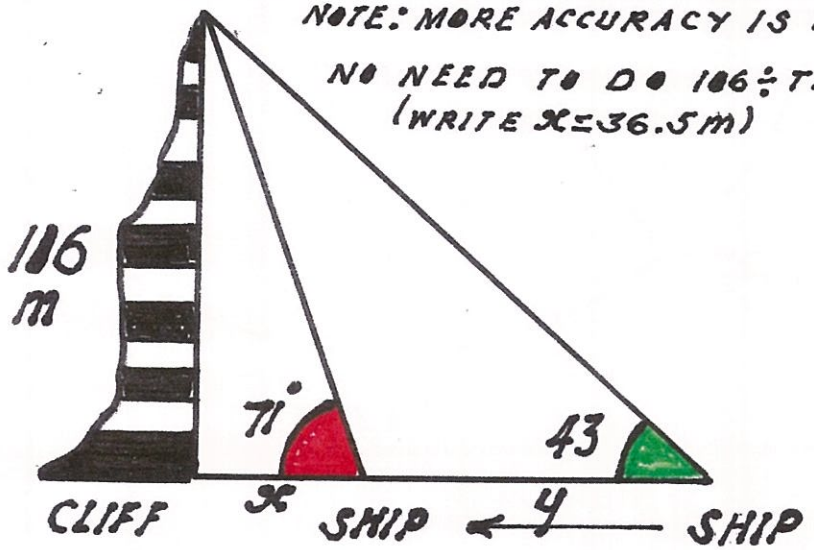
1193.



$$x + y = \frac{142}{TAN65} + \frac{142}{TAN37} \div 255$$

NOTE: MORE ACCURACY IS NOT REQUIRED HERE.

NO NEED TO DO 106 ÷ TAN 71 = ... LEAVE +/- + 106 ÷ TAN 43
(WRITE x = 36.5m)



$$x + y = \frac{106}{TAN43}$$

$$x = \frac{106}{TAN71} \div 36.5m$$

SUBTRACT $\therefore y \div 77m$

1094.

$SIN^2 53 + COS^2 53 = ONE$ (SEE 1092)

$$SIN\ x = \frac{17.86\ SIN 63}{25.13} = 17.86 \times SIN 63 \div 25.13 = \cdots$$

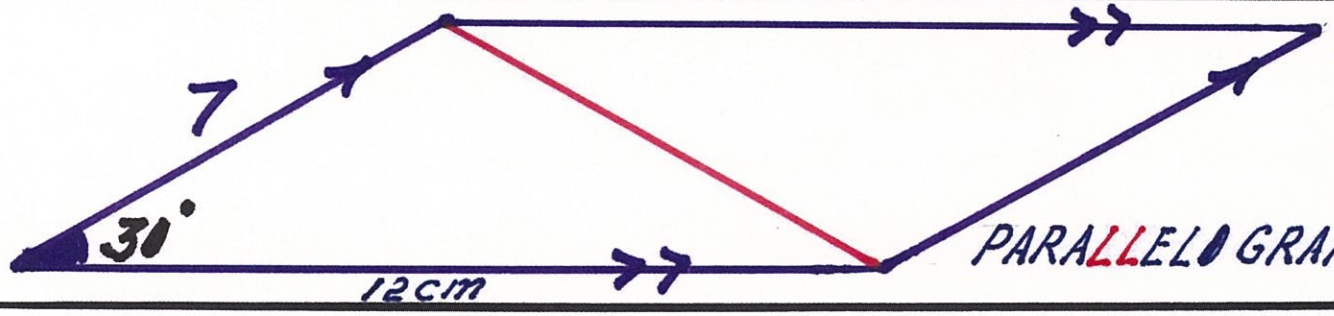
INV. SIN 39.5 $39.5^\circ = x$

$$\frac{x}{56.9} = \frac{SIN 37}{SIN 94}$$

TIMES - DIVIDE

A CROSS MULTIPLICATION SHORT CUT

$x \doteqdot 34.3$



2 CONGRUENT TRIANGLES

AREA

MENTALLY!

PARALLELOGRAM $84 SIN 30 = 42\ cm^2$

SEE 1090

RATIO SIGNS (±) IN THE 4 QUADRANTS | 1095.

VISUALISE THE "SIN-COS CLOCK"

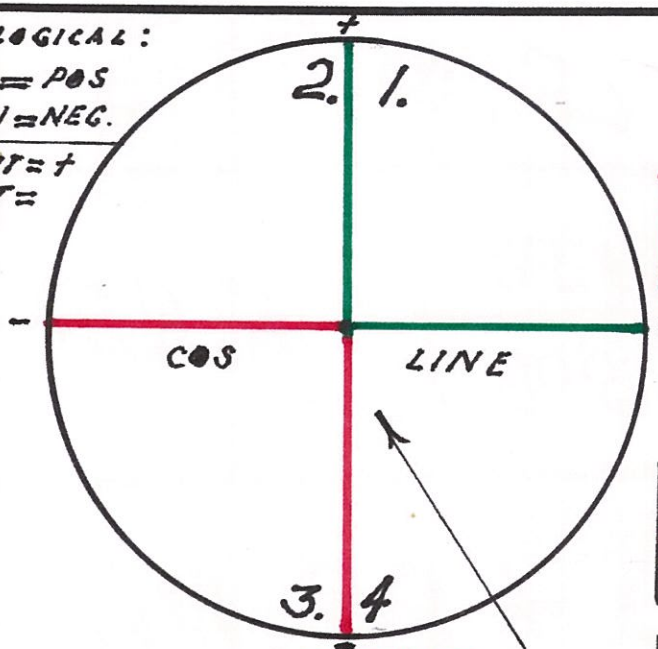
A NAME ONLY USED IN THIS PRACTICAL COURSE

DO NOT BOTHER MEMORISING THE TRADITIONAL FORMULAS !!
THEY DRIVE YOU MAD!

207

SO LOGICAL:
UP = POS
DOWN = NEG.

RIGHT = +
LEFT = NEG.



ONLY REMEMBER
SINE
LINE
∴

$-1 \leqslant$ SIN-& COS RATIOS $\leqslant 1$

SAME COLOUR: POSITIVE

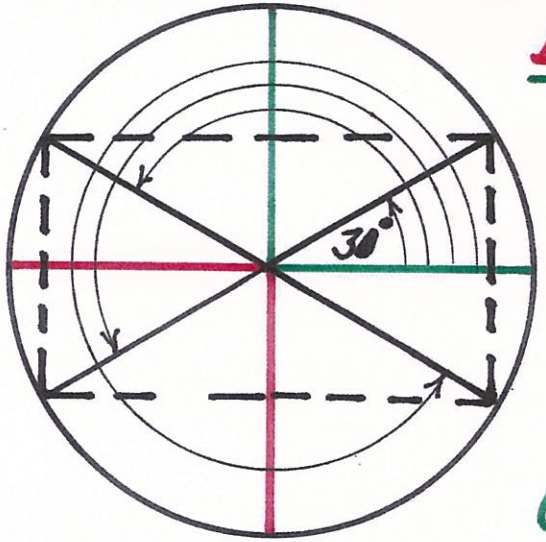
+COS LINE (BY ELIMINATION)

TAN $= \frac{SIN}{COS}$ AND THUS A COMBINATION

SIN: POS. IN 1 & 2. NEG. IN 3 & 4 (LOOK & SEE!)

COS & TAN: POS. 1 & 3. NEG. IN 2 & 4 DIFFERENT COLOURS

POSITIVE ANGLES: ANTICLOCKWISE



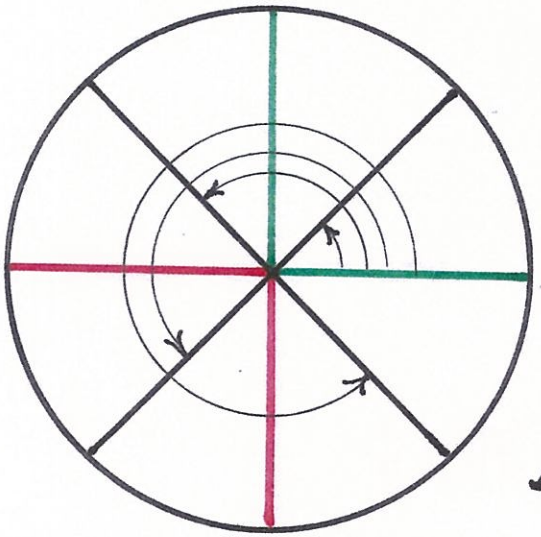
BASIC ANGLE 30°

$SIN\,30 = SIN\,150 = \frac{1}{2}$

$SIN\,210 = SIN\,330 = SIN\,-30 = -\frac{1}{2}$

$COS\,30 = COS\,330 = COS\,-30 = \frac{\sqrt{3}}{2}$

$COS\,150 = COS\,210 = -\frac{\sqrt{3}}{2}$



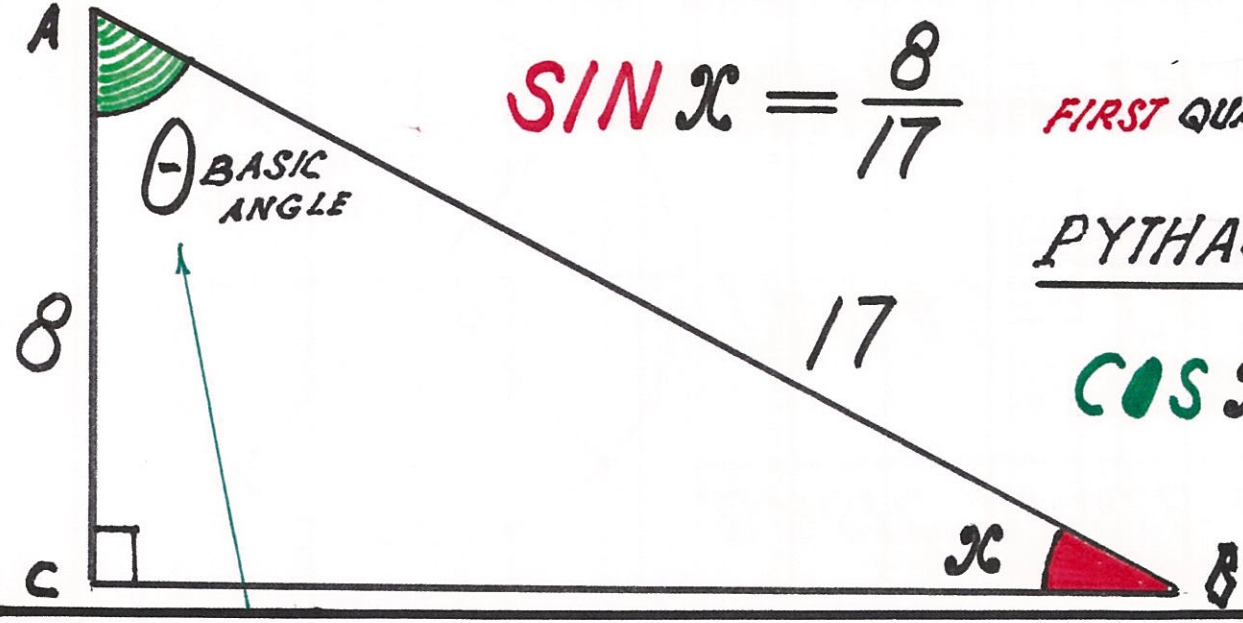
BASIC ANGLE 45°

TAN 45 = TAN 225 = ONE

TAN 135 = TAN 315 = NEG. ONE

-45

1097.



$\sin x = \frac{8}{17}$ FIRST QUADRANT (28°)

PYTHAGORAS TRIAD

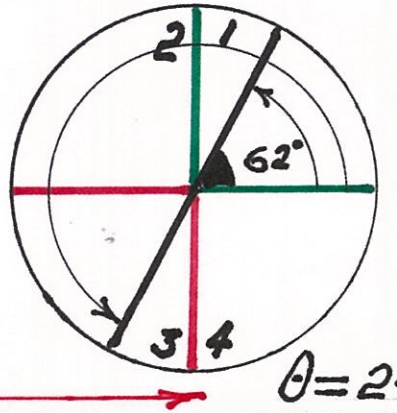
$\cos x = \frac{15}{17}$

$\tan x = \frac{8}{15}$

$\cos\theta = \frac{-8}{17}$ (QUADRANT 2 & 3. BASIC ANGLE 62°

$\tan\theta > 0$ (QUADRANT 1 & 3) $\frac{15}{8}$

$\sin\theta < 0$ (QUADRANT 3 & 4) $\frac{-15}{17}$



$\theta = 242^\circ$

FROM PARAMETRIC TO CARTESIAN

1098

$x = 2 + 3\cos\theta$ θ IS THE PARAMETER LINKING x & y

$y = 5 - 2\sin\theta$ SOLUTION: ELIMINATE θ

$\cos\theta = \frac{x-2}{3}$

$\sin\theta = \frac{5-y}{2}$

$\frac{(x-2)^2}{9} + \frac{(y-5)^2}{4} = 1$ IDENTITY (1092)

NOTE: $(5-y)^2 = (y-5)^2$

012

PHI

$x = 2\cos\phi$ $\quad y = 3\sin\phi$

$\cos^2\phi = \frac{x^2}{4}$ $\quad \sin^2\phi = \frac{y^2}{9}$

CARTESIAN EQUATION: $\boxed{\frac{x^2}{4} + \frac{y^2}{9} = 1}$

1099.

PROVE: $\frac{2\cos^2}{1-\sin} = 2+2\sin$ (SHORT HAND)

NOTE: x OR ϕ HAVE BEEN LEFT OUT FOR SIMPLICITY REASONS.

$$\frac{2\cos^2}{1-\sin} = \frac{2(1-\sin^2)}{1-\sin} = \frac{2(1-\sin)(1+\sin)}{1-\sin} = 2+2\sin \quad x \neq 91^\circ$$

$5\sin 15 = 5 \times \sin 15 \div 1.3$

$4\tan 41 \div 3.4$

$3\cos 75 \div .8$

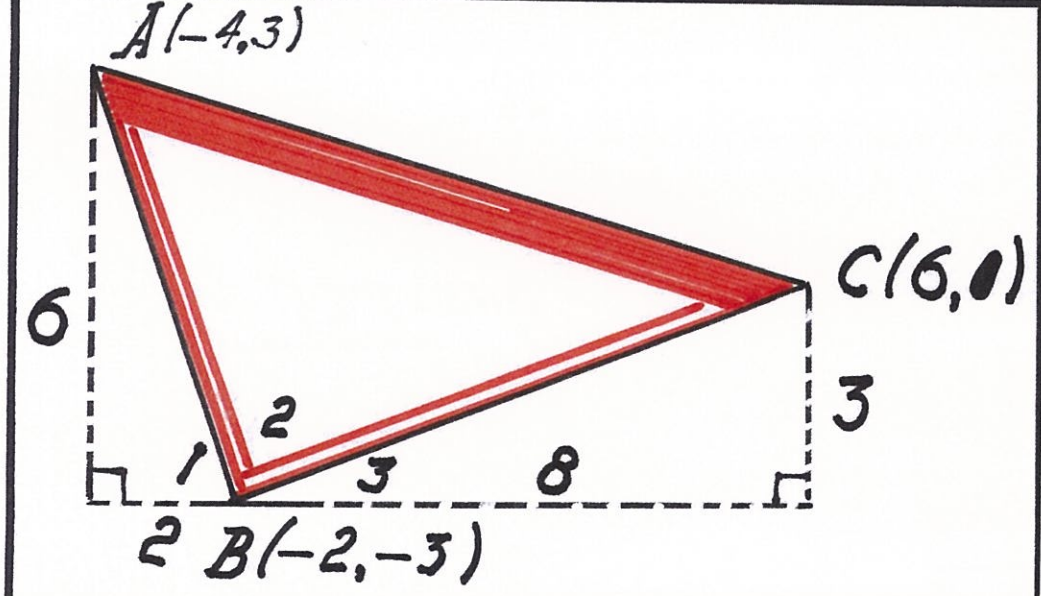
CONVERSIONS OF $\sin^2+\cos^2=1$

$$1 - 2\sin^2 x + \cos x = 0$$

$$1-\sin^2 - \sin^2 + \cos = \cos^2 - 1 + \cos^2 + \cos = \underset{\text{FACTORISE}}{2\cos^2+\cos-1=0}$$

$\therefore (2\cos-1)(\cos+1)=0 \therefore \cos x = \frac{1}{2} (x = 61^\circ \text{ OR } 311^\circ).\ \cos x = -1 (x = 180^\circ)$

212

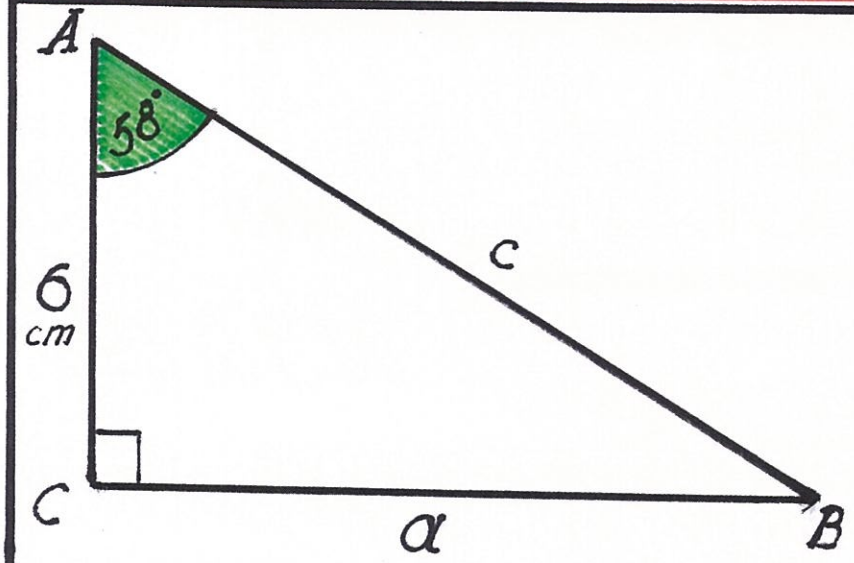


1100

$\tan B_1 = 3 \therefore B_1 \doteqdot 72^\circ$

$\tan B_3 = \frac{3}{8} \therefore B_3 \doteqdot 21^\circ$

$B_2 = 180 - 93 \doteqdot 87^\circ$



$a = 6 \tan 58 = 9.6\text{ cm}$

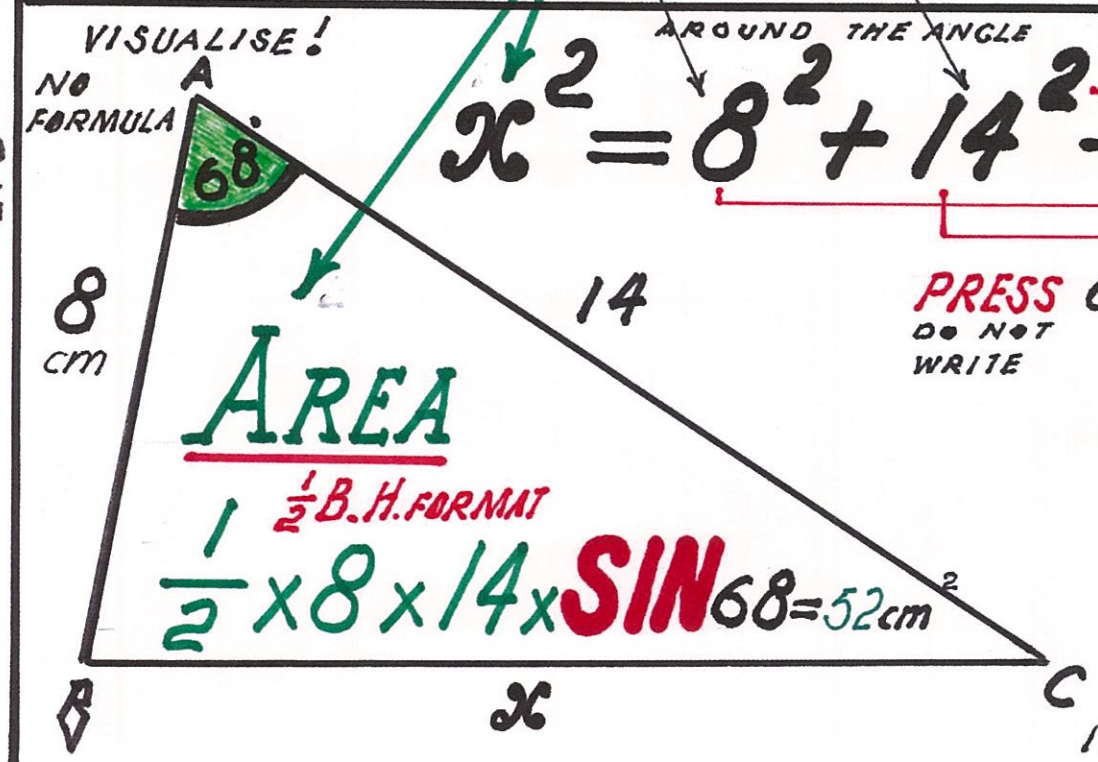
$c = \frac{6}{\cos 58} = 11.3\text{ cm}$

1. COS RULE PATTERN

1101.

GIVEN: S.A.S. SIDE - ANGLE - SIDE IN A NON-RIGHT TRIANGLE.

SEE HOW THE NUMBER PATTERN IS PUT TOGETHER.



AROUND THE ANGLE

DO NOT WRITE

$$x^2 = 8^2 + 14^2 - 2 \times 8 \times 14 \cos 68$$

WHILE LOOKING

PRESS $64+196-16\times14\times\cos68=\ldots$ DO NOT WRITE

REMEMBER

SQUARE ROOT

$x = 13.3$ cm

IT COULD NEVER BE 176 cm!

AREA

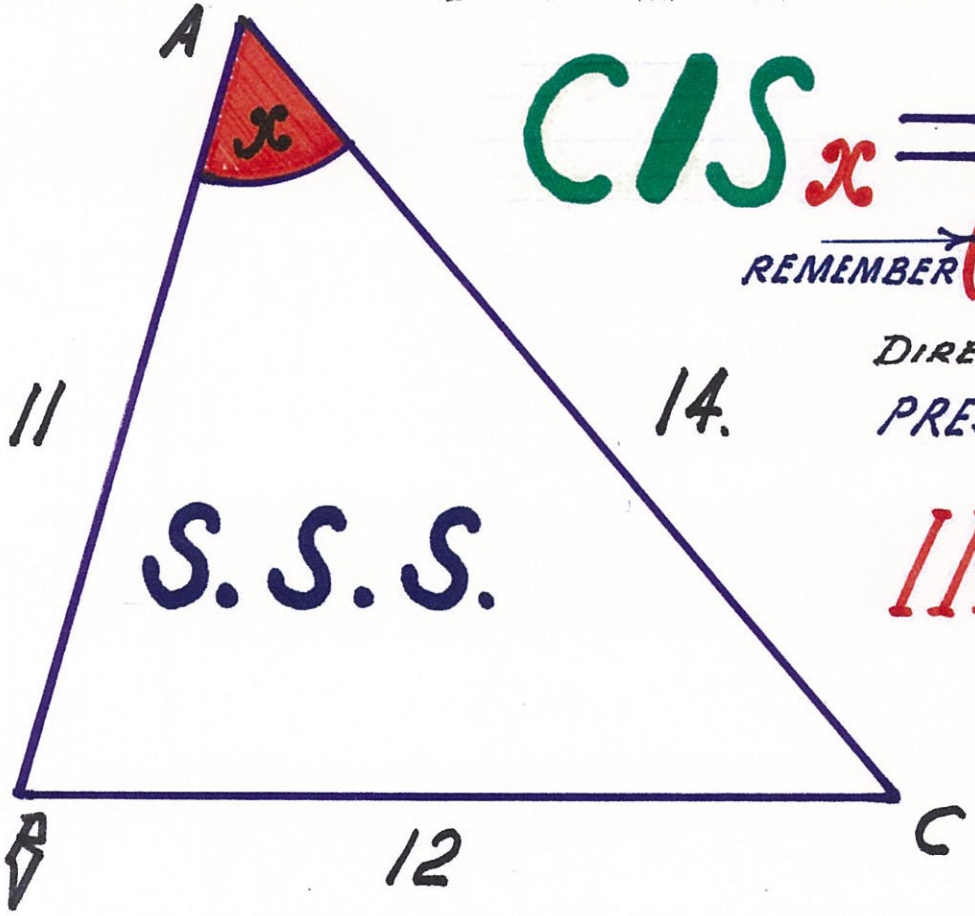
$\frac{1}{2}$ B.H. FORMAT

$\frac{1}{2} \times 8 \times 14 \times \text{SIN}\,68 = 52\text{cm}^2$

213

CONVERTED COS RULE PATTERN 1112.

THIS MUST BE VISUALISED, NOT WRITTEN!
DO NOT WRITE AN ABSTRACT FORMULA;
ESPECIALLY SINCE THERE ARE ONLY NUMBERS!



AROUND x

OPPOSITE x

$$\cos x = \frac{11^2 + 14^2 - 12^2}{2 \times 11 \times 14} = \ldots$$

REMEMBER

DIRECT BY CALCULATOR!

PRESS 121 + 196 − 144 = ... ÷ (22 × 14 = ...

INV COS

$x = 56°$

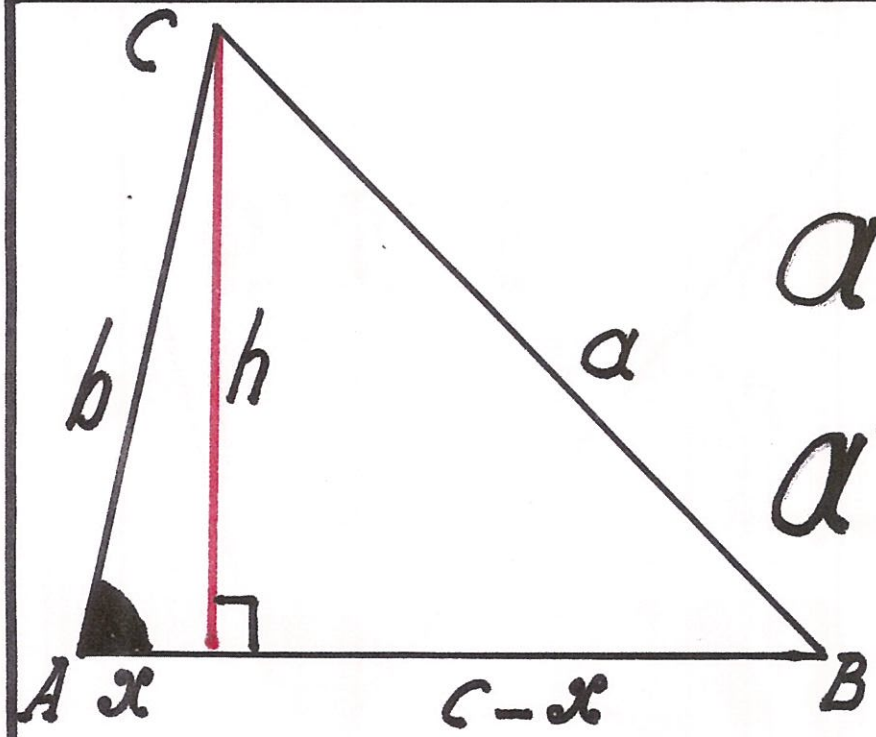
ANSWER ONLY

(LIKE MULTIPLE CHOICE)

2/4

PROOF COS RULE PATTERN

1113.



$$a^2 = h^2 + (c-x)^2$$

$$a^2 = h^2 + (c^2 - 2cx) + x^2 \quad (1)$$

$$x = b\cos A \;\&\; h^2 = b^2 - x^2$$

SUBSTITUTE (1) ∴

VISUALISE

$$a^2 = b^2 + c^2 - 2bc\cos A$$

THE OTHER SIDES — OPP. α

SIMILARLY:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

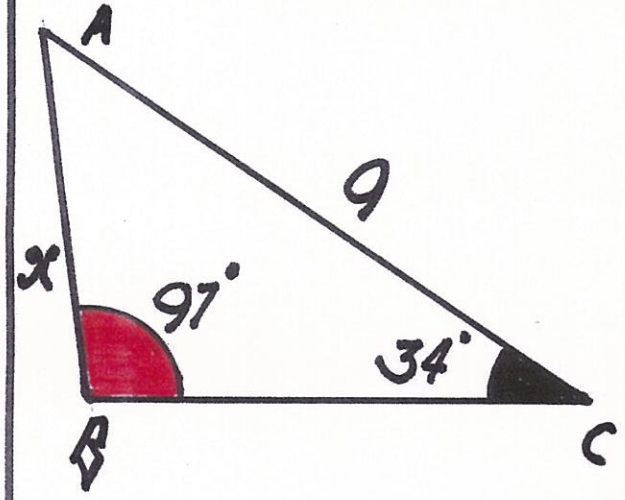
5/12 215

SINE RULE PATTERN

OTHER THAN S.S.S. | S.A.S.

1104.

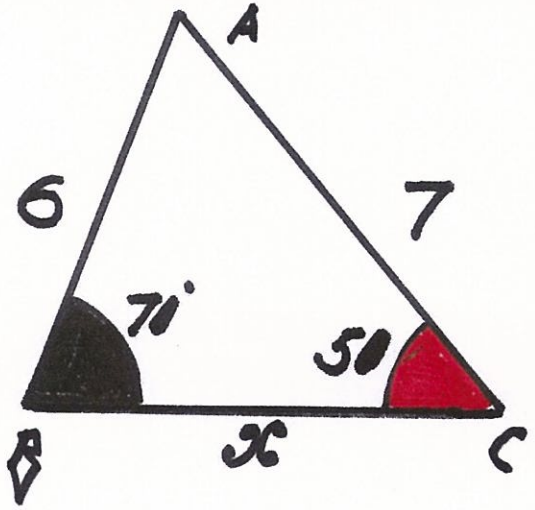
TYPE 1.



$$\frac{9}{\sin 97} = \frac{x \div 5}{\sin 34}$$

TIMES - DIVIDE

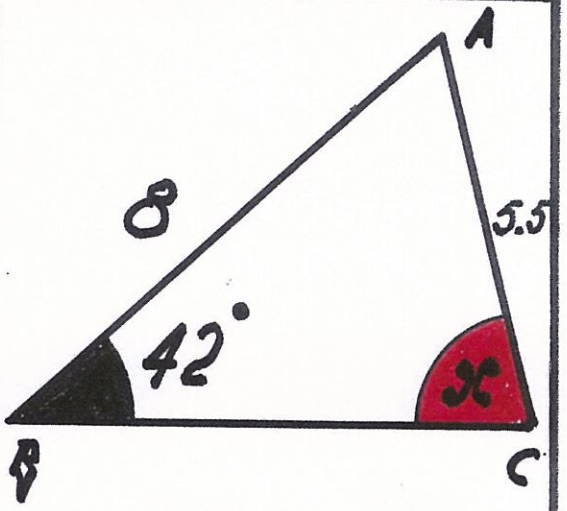
TYPE 2.



$$\frac{7}{\sin 71} = \frac{x \div 6.5}{\sin 61}$$

DO NOT USE THE RECIPROCAL VERSONS.
FOLLOW 'NATURAL' EYE MOVEMENT

TYPE 3.



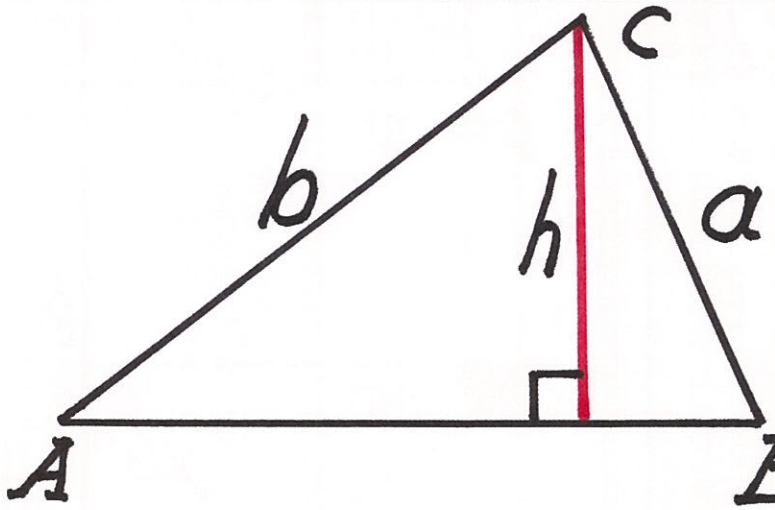
INV. SIN

$$\frac{5.5}{\sin 42} = \frac{8}{\sin x}$$

$x \div 77°$

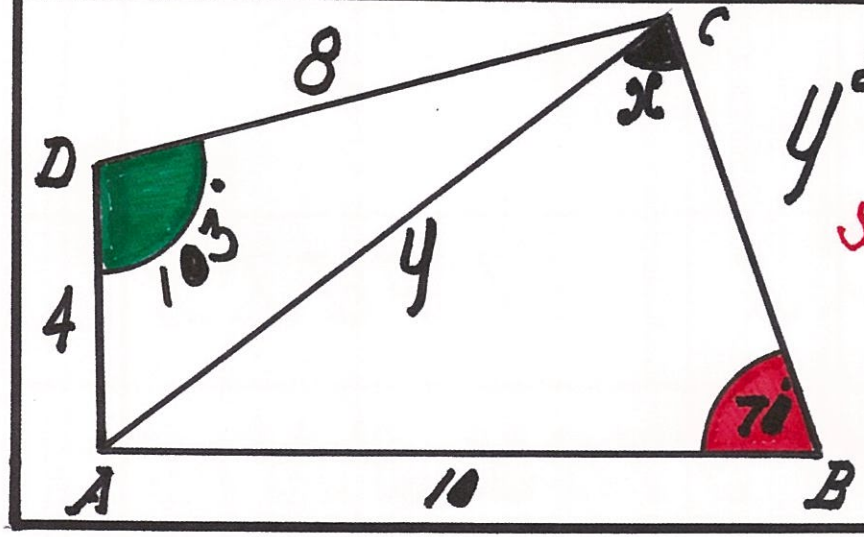
912

PROOF SINE RULE PATTERN 1105



$$h = a\sin\hat{B}$$
$$h = b\sin A$$
$$\Big\} \frac{a}{\sin A} = \frac{b}{\sin B}$$

DO NOT ADOPT THE RECIPROCAL FORMAT.!
IT LOOKS TOP HEAVY, SOUNDS AWFUL & UNRHYTHMIC.



$$y^2 = 4^2 + 8^2 - 2\times4\times8\cos113 = \ldots$$

SQUARE ROOT ... PRESS M_{IN} (WRITE $y \doteq 9.7$)

$$\left(\frac{M_R}{\sin 71} = \frac{10}{\sin x}\right)$$

DIVIDE
TIMES
INV. SIN $x \doteq 75°$
MEMORY BUTTON

112

TRIGONOMETRY: TASK 1.

202 $\hat{A}=70^\circ$, $\hat{C}=104^\circ$, $\hat{D}=120^\circ$, FIND x

FIND A_1 & A_2 BY USING THE SINE RULE (195)

DRAW THE QUADRILATERAL TO MEASURE AC

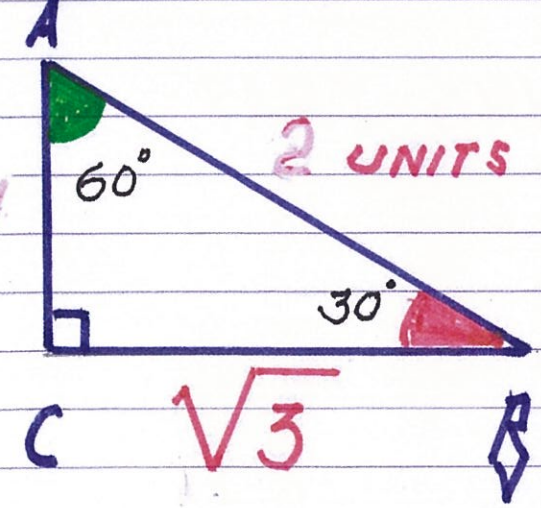
204 MEMORISE THE 4 IDENTITIES

205 BASE ANGLES 70° & 50°

$H=9$ CALCULATE x & y

206 $\sin^2 30 + \cos^2 30$

$\frac{x}{6} = \frac{\sin 40}{\sin 80}$



PYTHAGORAS

$\sin 30^\circ = \cos 60^\circ = \frac{1}{2}$

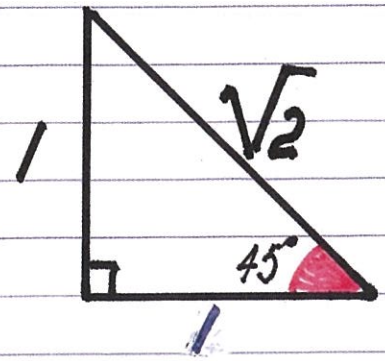
BUT NOT BY CALCULATOR!

$\sin 60^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}$

$\tan 30^\circ = \frac{1}{\sqrt{3}}$

$\tan 60^\circ = \sqrt{3}$

TRIGONOMETRY: TASK 2.



$$\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\tan 45^\circ = 1$$

209

RIGHT TRIANGLE:

$\overline{AC} = 9$, $\overline{AB} = 41$ FIND $\hat{B}$

210

$x = 3 + 4\cos\theta$ | $y = 7 - 3\sin\theta$

$x = 3\cos\phi$ | $y = 5\sin\phi$

211

PRACTISE PROVING THAT

$$\frac{2\cos^2}{1-\sin} = 2 + 2\sin$$

212

PLOT A(-6,5), B(-3,-3) C(4,0) | CALCULATE $\hat{B}2$

TRIGONOMETRY: TASK 3

212 $\hat{A} = 63^\circ$, $\overline{AC} = 8$. FIND a & c

213 $\overline{AB} = 10$, $\overline{AC} = 16$, $\hat{A} = 70^\circ$ AREA & $\mathcal{R}$

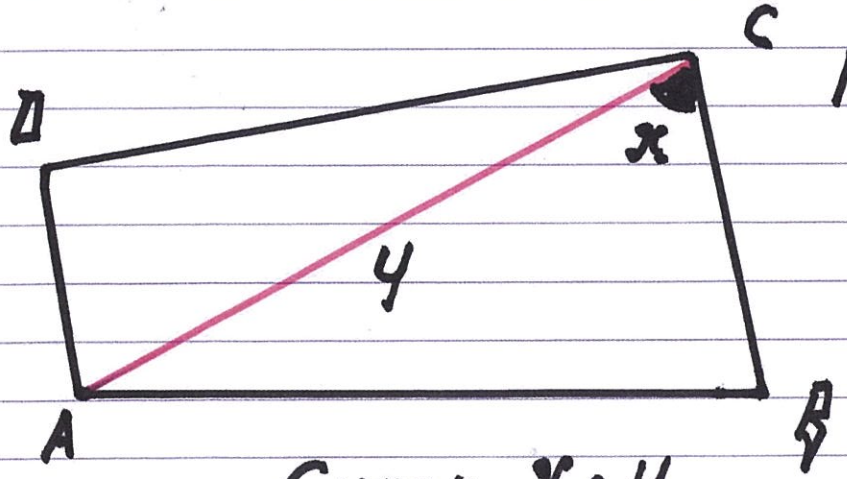
214 $\overline{AB} = 13$, $\overline{AC} = 17$, $\overline{BC} = 15$ FIND x

215 PRACTISE THE PROOF

216

$\hat{B} = 102^\circ$	$\hat{B} = 65^\circ$	$\hat{B} = 40^\circ$
$\hat{C} = 40^\circ$	$\hat{C} = 40^\circ$	$\overline{AB} = 10$
$\overline{AC} = 11$	$\overline{AB} = 8$, $\overline{AC} = 9$	$\overline{AC} = 6$

217 PRACTISE THE SINE RULE PROOF.



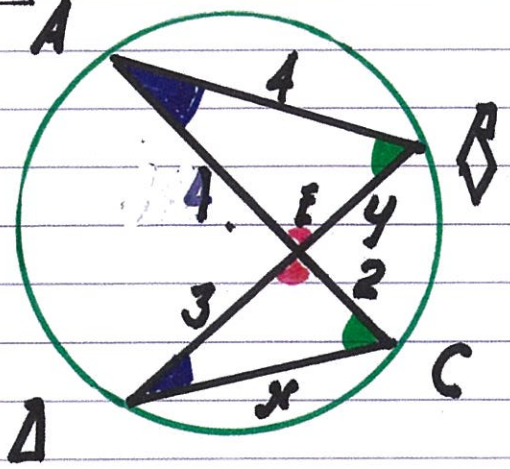
MEASURE SIDES & ANGLES

CHECK x & y

ANSWERS

1 $4x=60^\circ$	$x=30^\circ$	VERTICAL ANGLE 80°
2 360°	3 $x=90^\circ$	4 $5\times180^\circ$
5 72° & 108	6 60° & 120°	7 45° & 135°
10 $x+y=50^\circ$	$x=140^\circ$	11 $B_1=70^\circ$, $D_1=50^\circ=x$ $y=80^\circ$
11 $30^\circ, 60^\circ, 90^\circ$	15 $A=116\ cm^2$ $P=58\ cm$	16 $4H=20 \therefore H=5$ cm $P=30$ cm
16 $P=29+6\sqrt{2}$ cm	17 $(12\times18)+(2\times12)$ $A=240 cm^2$	18 $C=10\pi$ cm $A=25\pi\ cm^2$ 19
21 $A=\frac{1}{9}\times16\pi\ cm^2$ $P=\frac{1}{9}\times8\pi$ cm	$A=\frac{1}{3}\times36\pi=12\pi\ cm^2$ $P=\frac{1}{3}\times12\pi=4\pi$ cm	22 $A=36\pi-16\pi=20\pi$ $P=12\pi+8\pi=20\pi$ cm
23 $A=\frac{1}{3}\times20\pi\ cm^2$ $P=\frac{1}{3}\times20\pi$ cm	24 $A=22^2-50\pi\ cm^2$ $P=(8\times6)+20\pi$ cm	25 $A=(2\times24)+(64\pi-36\pi)=$ $48+28\pi$, $P=48+4\pi$ cm
26 $16\ cm^2$	$\frac{1}{2}\times90\ SIN\ 88$	$A=\sqrt{10\times5\times2\times3}=$ $\sqrt{300}$
28 $A=28\ SIN\ 30$ $\doteq 14$	31 $12x=360^\circ$ $x=30^\circ$	$\hat{A}+\hat{C}=180^\circ$
$II+O+V+III=$ $II+V+O+III$ $\therefore AD+BC=AB+CD$	DRAW ABCD	

33



$\triangle AEB \sim \triangle DEC$

LETTERS MUST CORRESPOND!

$$\frac{y}{2} = \frac{4}{3}, \quad y = 2\frac{2}{3}$$

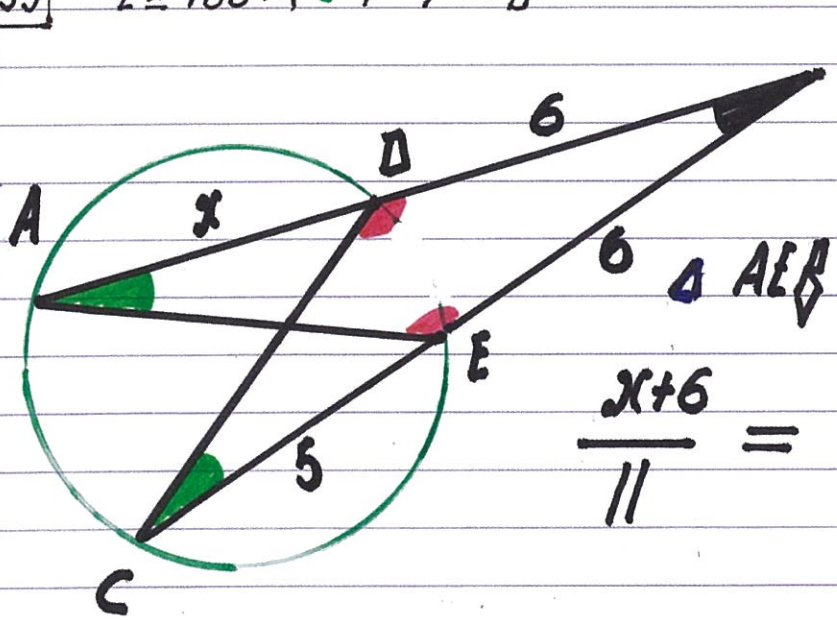
$$\frac{x}{4} = \frac{3}{4}, \quad x = 3$$

31

$\overline{CD} = 10$

$AE = 5$

35 $\hat{E} = 180° - (\bullet + \bullet) = \hat{D}$



$\triangle AEB \sim \triangle CEB$

$$\frac{x+6}{11} = \frac{6}{6}, \quad x = 5$$

39

$x^2 + y^2 = 64$

40

$(x-4)^2 + (y-2)^2 = 64$

53 $4 = \frac{1}{x} \therefore x = 1/4$

$4x + 10x = 10x + 15$
$x = 3\,3/4$

54 $12x - 6 = 1 + 5x - 5$
$x = 2/7$

$3x + 1 = 30x + 10x - 20$
$x = 1\,21/37$

55 $10x + 5 = 12x - 2$
$x = 3\,1/2$

$x - 3 = 18$
$x = 21$

56 $56x + 28 - 21x = 12x + 36$
$x = 4/11$

58 $18(2x-1) - 8(2x-4) = 15$
$x = 1/20$

59 $x = 1\,1/4, \; x = 1\,2/5$

60 $10x + 4 = 44, \; x = 4$
SIDES 8, 13, 23

61 $20 + 2x = 60, x = 20$ $A = 200\,cm^2$

62 1 CAT 108 DAYS
9 CATS 12 DAYS

63 $9x - 7 = 3x + 5, \; x = 2$
$y + 2 = 18 - y, \; y = 8$

64 $C + 1/4\,C + \frac{1}{12}C = 16$
$16\,C = 192 \therefore C = 12, B = 3, A = 1$

65 AREA $(2x+3)(4x-2)$
$\therefore 12x + 2 = 38 \therefore x = 3, A = 90\,cm^2$

66 $(x-5)$ & $(x+10)$

$4(B-8) = A - 8 \therefore 4B - 24 = A$
NOW $3B = A \therefore 4B - 24 = 3B \therefore B = 24$
$A = 72$

ANGLES 30°, 60° & 90°

67 $6H = 180 \therefore H = 30$
$B = 60$

$B + H = 50\,cm$
$B = \frac{3}{5}$ of $50 = 30, \; H = 20\,cm$

71 $(5a-3)(25a^2+15a+9)$

72 $(5a+3)(25a^2-15a+9)$

73 x^2+5x+6

x^2-x-12

74 $x^2-12x+35$

$6x^2+4x-10$

75 $(x-3)(x-5)$

76 $(x-6)(x+4)$

78 $(15x-6)(x+1)$

80 $(x-2)(3x-1)$

81 $(x+3)(x-3)=x^2-9$

$b(a^2+c-d)+ac$

82 $\frac{x}{x+3}$ $x \neq 3$

$\frac{x+2}{x+4}$ $x \neq 4$ 83

85 IF 3% OF $x = 60$, $x = 60 \div .03 = 6000 \div 3$

86 DEPOSIT: $\frac{1}{4} \times 18000$ + \$14000 = \$18500.
\$500 MORE ON \$13500 IN 20 MONTHS WHICH IS
$500 \div 20 \times 12 = \$300$ IN 1 YEAR WHICH IS
$300 \div 135 \doteq 2.2\%$

87 $(30 \times 20) + (7 \times 40) + (5 \times 30)$

89 $\$28000 \div 13 \times 1.18 = 2542$

89 $(8 \times 18) + (2 \times 27) + (2 \times 36)$

90 $18 \times 1.15 \times 8 = \$165.60$
PUT ↑

90 $(4 \times 24) + (3 \times 32) + (7 \times 32)$

91 $(30 \times 12) = \$360$

91 $4 \times 7 \times 52$

92 $200 + 8 \times 30$

92 $\$329.80$

PAY $\$400 \times .85 \times .97$

93 $25 \times 12 \times 50 = \cdots - 50000 = \cdots$
$\div 25 = \cdots \div 500 = 8\%$

94 $12 \times 20 \times 8 \times 15 = \cdots$ 11.5%
$- 15000 = \cdots \div 8 = \cdots \div 150$

95 DEP. $1/5 \times 700 = \$140$ BALANCE $\$560$
REPAY $2.16 \times 560 = \$1209.6$, $\$50.40$ PM
INTEREST PAID $.16 \times 560 = \$89.60$

97 $9000 \times 1.9 = \cdots \div 60 = \$285$ P.M

$60 \times 220 = 13200 - 9000 = \$4200$
$\div 5 = 840 \div 90 = 9.3\%$

98 PAID $264x$. INTEREST $1260x$
PRINCIPAL $138x = 9000$. $x \div 6.5$ YEARS

$2.28x\,(100\% + 128\%) = 72 \times 200 = \cdots$
$\div 2.28$ $\therefore x \div \$6316$

99 DO: $8 \div 20$, $15 \div 35$, $30 \div 65$.
$45 \div 100$ (.4, .43, .46, .45)

100 $\$172.55$

101 $18 \times 145 = \cdots \div 60 = \$435$ P.M.

$2000 \times 1.16 = \cdots \div 24 = \$96.67$ P.M.

104 16000×1.04^4 IN 4 YEARS

120000×1.08^6

105 $\frac{10}{14} = \frac{x}{500}$, $14x = 5000$
$x = 357$

106 1.12×24

107 $234 \times .85 = \cdots$
$\times 1.1$

105 MENTALLY
$56 \div 2.8 = 560 \div 28 =$
20%

106 $.86 \times 48$

107 $.95x = 120$
$x = 120 \div .95$

120 $(x+3)(x-3)=0$

$x=\pm 3$

$x=0$

$x=-12$

121 $x=3/4$

$x=-2/7$

122 $x=2$

$x=-1/3$

129 WRITE AS $x^2+12x-7=0$

$x^2-14x+5=0$

130 $x+10x+25$

$(x+5)^2$

$x^2-16x+8$

$(x-8)^2$

133 EXPAND

$Ax^2+4Ax+4A+Bx-B+C=0$ (FACTORISE

$Ax^2+(4A+B)x+(4A-B-C)=0$

5 $\quad -3 \quad 7 \quad \therefore 20+17+C=7 \therefore C=30$

$\therefore B=-17$

ANSWER: $5(x+2)^2-17(x-1)+30=0$

138 $(x-4)(x+5)=0$

$x^2+x-20=0$

$(x+3)(x-7)=0$

$x^2-4x-21=0$

140 $(\alpha+\beta)=2 \; ; \; \alpha\beta=2/3$

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{(\alpha+\beta)}{\alpha\beta}=\frac{2}{2/3}=3$

$(\alpha+1)(\beta+1)=\alpha\beta+(\alpha+\beta)+1=3\,2/3$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=4-4/3=2\,2/3$

141 $\alpha+\beta=\frac{-k}{4} \therefore \beta=\frac{-k}{4}-2$

$\alpha\beta=\frac{7}{4} \therefore \beta=\frac{7}{4}\div 2=\frac{7}{8}$

$\left.\begin{matrix}\frac{7}{8}=\frac{-k}{4}-2\end{matrix}\right\}$

$7=-2k-16 \therefore 2k=-23 \therefore K=-11\,1/2$

143

$\Delta>0$	$\Delta=0$	$\Delta<0$
ROOTS	EQUAL ROOTS	NO ROOTS

146 $a=16,\ d=4$

TEST $\frac{1}{2}(16+24)=20$ ✓

$T_1=1,\ T_2=5,\ T_3=4\times3-3=9$

$d=4,\ \frac{1}{2}(1+9)=5$ 147

$S_{30}=\frac{1}{2}\times15\times62=930$ 147

$T_{24}=6+23\times4=98$ 148

$T_7=31$ 150

$T_5=4^5=1024$ 150

151 $a+9d=28$ SUBTRACT $6d=18$

$a+3d=10$ $d=3,\ a=1$

$a^6\div a^2=a^4=128\div8=16$

$r=2$ $a=8\div4=2$

152 $T_n=6+(n-1)4=30$

$6+4n-4=30,\ n=7$

LOG USES BASE 10

ln USES BASE e

152 THE USE OF LOGARITHM,

$\log_{10}10=1$ BECAUSE $10^1=10$

NOT SHOWN → $_{10}$

$\therefore \log_{10}1000=3$. BECAUSE $10^3=1000$

NOT SHOWN → $_{10}$

$T_n=ar^{n-1}=2\times3^{n-1}=486$

$3^{n-1}=243$. TO SOLVE, USE LOG OR ln

WATCH! $(n-1)\log3=\log243$

$(n-1)=\log243\div\log3$

NEW CASIO: $\log243=\div\log3=5$

$\therefore n=6$

153 $3+(n-1)3 > 130$

$n > 127 \div 3 + 1 = 43.3 \therefore T_{44} > 130$

$2 \times 2^{n-1} > 176 \therefore 2^{n-1} > 88$

$(n-1)\ln 88 = \ldots \div \ln 2 = 6.5 \therefore n > 8$

154 $T_1 = 3, r = 2$

TEST: $3 \times 12 = 6^2$

155

$4 \times 36 = 12^2 = 144$

164

$SA = 64\pi$ cm²

$V = \frac{1}{3} \times 64\pi$

165 **CONE** $V = \frac{1}{3}BH = 8 \times 49\pi$

$SA = 49\pi + (25 \times 7\pi) = 224\pi$ cm²

166 SLANT EGE 5 CM (TRIAD)

$V = \frac{1}{3} \times 36 \times 4 = 48$ cm³. $SA = 96$ cm²

167 $V = 36\pi \times 8 = 228\pi$ cm³

$SA = 36\pi + 96\pi = 132\pi$ cm²

168 $SA = 1:9$, $V = 1:27$

168 $V = 4\pi \times 2 = 8\pi$ m³

$= 8000\pi$ L

VOLUME RAIN: $36 \times .008$ m³

BASE TANK π m². RISE: $2.88 \div \pi =$ 92 mm

168 $2R\pi = 18$; $R = 9 \div \pi = \ldots$ SQ = ...

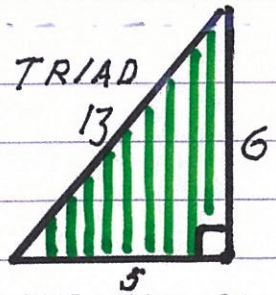
$\times \pi = \ldots \times 30 = 773.5$ cm³

169 $V = 210$ cm³.

$SA = (2 \times 50) + (2 \times 42) + (2 \times 35)$ cm²

170 SIMILAR PYRAMIDS: BASE TOP = 5 cm $\therefore V = 50$ cm³, $V_{TOP} = 400$ cm³

SIDES $1:2 \therefore V\ 1:8$ ✓



WHOLE SA $= 4 \times 65 + 100 = 360$ cm²

$h^2 = 36 + 6.25 = 42.25$

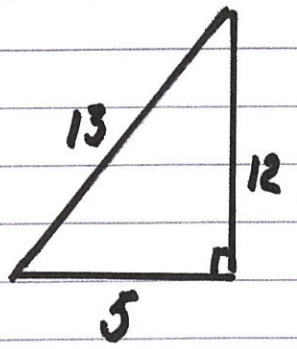
$h = \sqrt{42.25}$

SA 4 $\triangle$ = $4 \times 2.5 \times \sqrt{4}$...

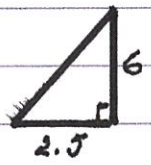
SA FRUSTRUM: $360 - 10\sqrt{42.25} + 25$ TOP

cm²

171



$V_{TOP}: 12.5\ cm$



VOLUME

FRUSTRUM

$V_{WHOLE}: 100\pi\ cm^3$

$87.5\ cm^3$

SA WHOLE $25\pi + 60\pi = 85\pi\ cm^2$

SA TOP - WITHOUT BASE $= 15\pi\ cm^2$

SA FRUSTRUM $70\pi + 6.25\pi = 76.25\pi\ cm^2$

172 GRAVEL AREA $80 - 24 = 56\ m^2$

VOLUME $56 \times .04 = 2.24\ m^3$

$2R \times \frac{22}{7} = 88$, $R = 2cm$. $V = 40\pi\ cm^3$. $SA = 40\pi\ cm^2$

174 $\tan\alpha = \frac{3}{4}$ LINE LEANING LEFT

m = NEGATIVE $\frac{3}{4}$

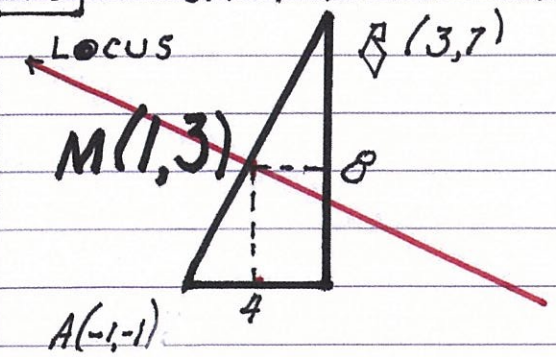
$x = 1.5$ | $y = -1$ | $y = 0$, $x = 0$

175 $y = 4x - 7$ | $m = \tan 50 = 1$ $y = x + 3$

$y = -\frac{2}{5}x$ | $6 + b = 5$ $y = 3x + 5$

$15 + b = 8$, $b = -7$, $y = 5x - 7$

176 ONLY DRAW WHAT'S NECESSARY!



$m\overline{AB} = 2$

$\therefore$

$m\,\text{LOCUS} = -\frac{1}{2}$

MID POINT (1,3) SUBSTITUTE IN $mx+b=y$ TWICE

1. $\overline{AB}$: $2+b=3$, $b=1$. EQUATION $\overline{AB}$: $y=2x+1$

2. LOCUS: $-\frac{1}{2}+b=3$, EQUATION: $y=-\frac{1}{2}+3\frac{1}{2}$

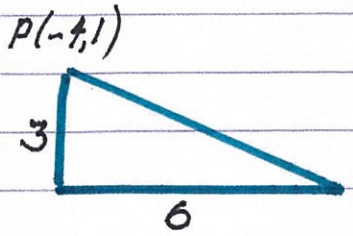
177

ONLY LEARN ONE METHOD!

$9x+3y=12$ (1) ADJUSTED

$+\quad 4x-3y=14$ (2) $Q(2,-2)$

$13x = 26$, $x=2$, SUB(1) $y=-2$



$m\overline{PQ} = -\frac{1}{2}$

USING Q (SUB. IN $mx+b=y$: $-1+b=-2 \therefore b=-1$

EQUATION PQ: $y=-\frac{1}{2}x-1$

187

$$3x+2y=4 \quad (2)$$

$$-\ 3x-3y=6 \quad (1)$$

$$5y=10, \quad y=2, \text{ SUB (1) } x=4 \qquad Q$$

LINE $\parallel$ $-2x+y-15=0$

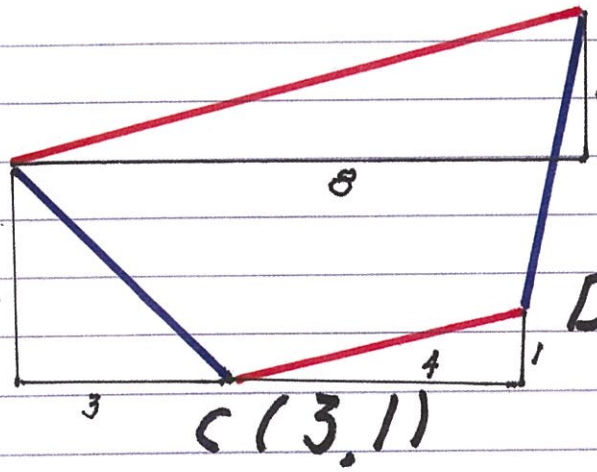
WRITE AS $y=2x-5$ $\quad m=2$

USING $Q(4,2)$ SUB $mx+b=y$

$$8+b=2, \quad b=-6$$

EQUATION LINE IN QUESTION $y=2x-6$

179



$$m\overleftrightarrow{AB}=m\overleftrightarrow{CD}=\frac{1}{2} \quad \therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$$

$\therefore$ ABCD IS A TRAPEZIUM

180

$Ax+By+C$

$2x-7y-14=0$

x-INTERCEPT (0,-2)

y-INTERCEPT (7,0)

P(-2,8)

MEASURE

y

x

(7,0)

(0,-2)

$\varphi = 11°$

$2x-7y-14$ (A, B)

SUB (-2,8)

$-4-56-14$

$|-74|$ ← ABSOLUTE VALUE = 74

$D = 74 \div \sqrt{A^2+B^2} \div 10$

181

$6 \div \sqrt{4+9} = 6 \div \sqrt{13} = 1.7$

$9 \div \sqrt{13} = 2.5$

2.5 MEASURE

B(-3,1

2

3

A CLOSER B

$2x+3y-6$ (A, B)

SUB (6,-4) 12 -12-6 |-6| 6

SUB (-3,1) -6+3-6 |-9| 9

$3y = 2x+6$

1.7 MEASURE

A(6,-4)

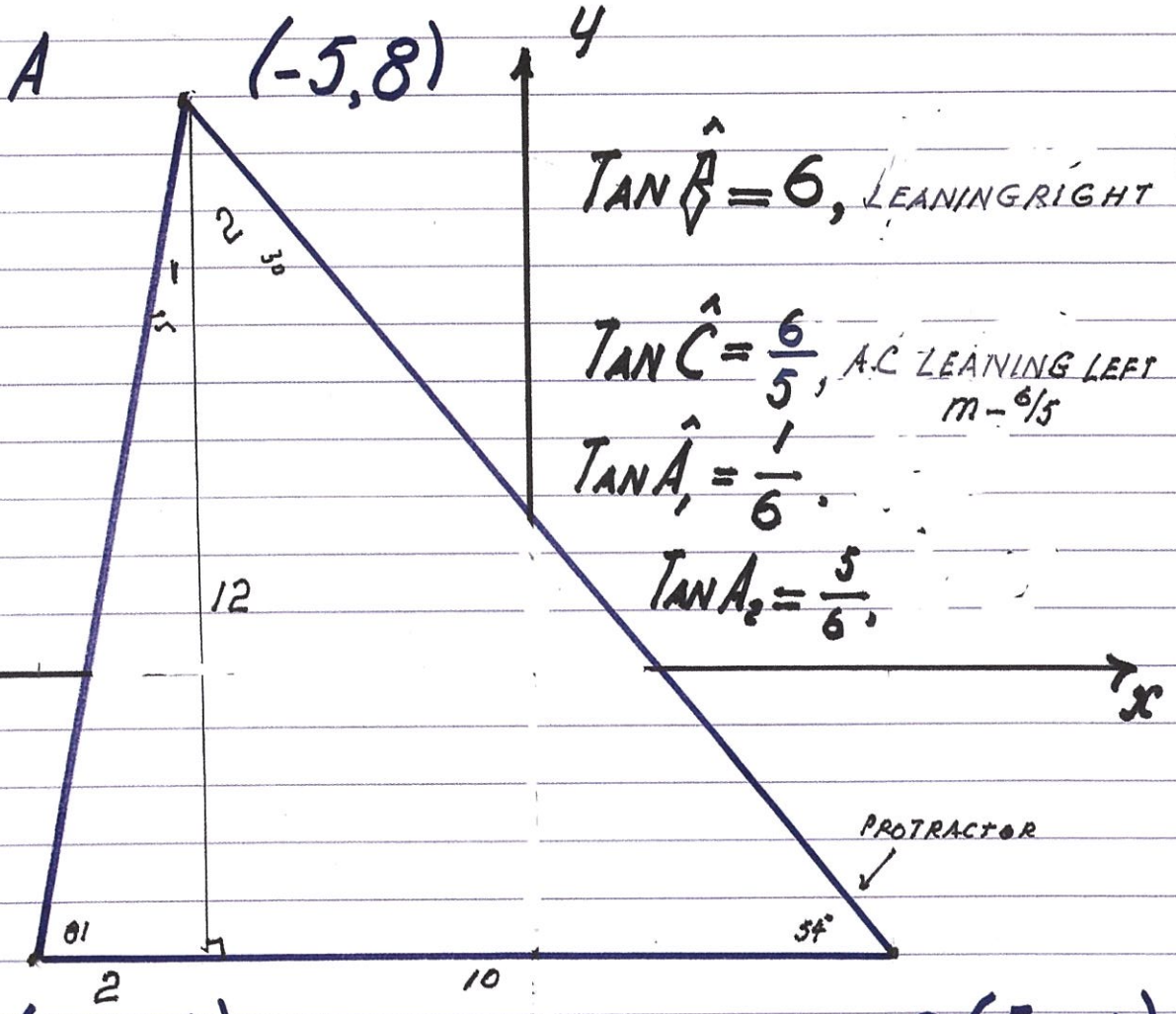
182

$mx+b=y$ SUB $(-6,-2)$

$-12+b=-2$

$b=10$

LINEAR EQUATION $y=2x+10$



$\tan\hat{B}=6$, LEANING RIGHT

$\tan\hat{C}=\frac{6}{5}$, AC LEANING LEFT $m=-6/5$

$\tan\hat{A}_1=\frac{1}{6}$.

$\tan A_2=\frac{5}{6}$.

$B(-7,-4)$, $m=6$

$-42+b=-4$, $b=38$

EQ $y=6x+38$ IS A ON IT?

SUB A: $8=-30+38$ ✓

$mx+b=y$ $C(5,-4)$

$-\frac{6}{5}\times 5+b=-4$ $m=-6/5$

$b=2$

EQ AC: $y=-\frac{6}{5}x+2$. SUB A: ✓

187 | 30 | $2\sqrt{7}$ | $6\sqrt{6}$ | $7\sqrt{7}$

188 | $\sqrt{3} \div 3$ | 4

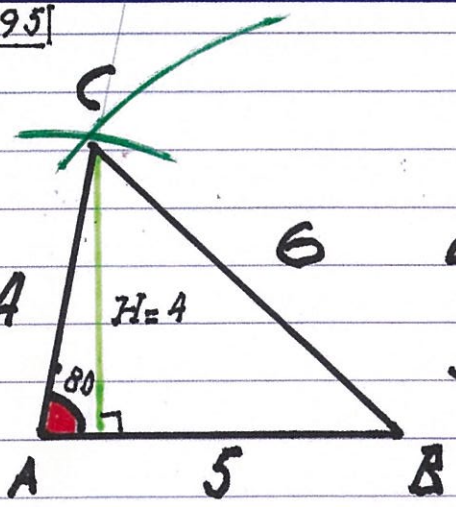
189 | $t^2 = 18$ | 278 | $3 \leq 7$

190 | $5a^3$ | 21 | $5\sqrt{5} + 3\sqrt{7} - \sqrt{7} = 5\sqrt{5} + 2\sqrt{7}$

191 | 12

$9\sqrt{2}$ | $\sqrt{a} = 9\sqrt{2}, \; a = 162$

195



$P = 15$ cm

1. $A = 10$ cm^2

COMPARE

2. $A = 10 \sin 80 = 9.5$ cm^2

COMPARE

3. $A = \sqrt{7.5 \times 3.5 \times 2.5 \times 1.5} = 9.92$

196

AREA $\triangle ACD = 12$ cm^2

AREA $\triangle BCD = 24$ cm^2

} 1 : 2

SIDES 1 : 4

AREAS 1 : 16

$x = \sqrt{39} \doteq 6.2$ | $x = \sqrt{56} \doteq 7.5$

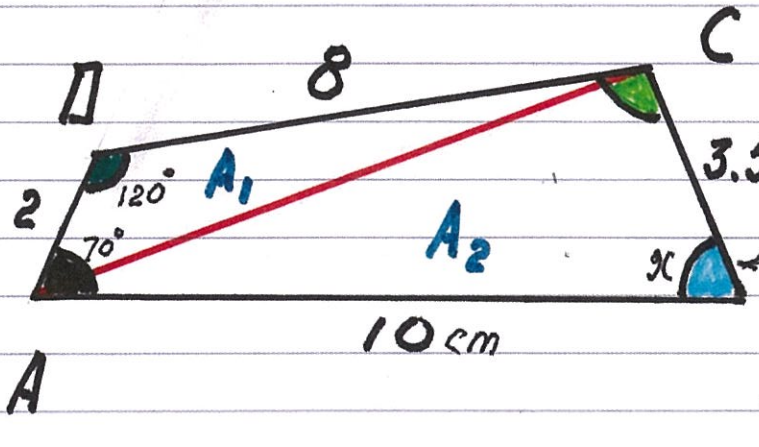
198 | $\frac{x}{8} = \frac{5}{3}, \; x = 13\frac{1}{3}$ | $y = 10.8$ | $x = 9, AC = 15$ | SA 9 : 49 | V 27 : 343 | 1 : 4

202

$x = 66°$

$AC = 9.2$ cm

A	A_2
$8 \sin 120$	$17.5 \sin 66$



DISCREPANCIES
- HUMAN ERROR
- LINE THICKNESS

CHECK (NOT QUITE

205

$x = 9/\tan 70$

$y = 9/\tan 50$

206

ONE

$x = 6 \sin 40 \div \sin 80$

209

TRIAD

9.40.41

$\tan \hat{B} = 9/40$

$\sin \hat{B} = 9/41$

$\cos \hat{B} = 40/41$

REMEMBER: UPSIDE DOWN T TAN

2/3 LETTER N SIN

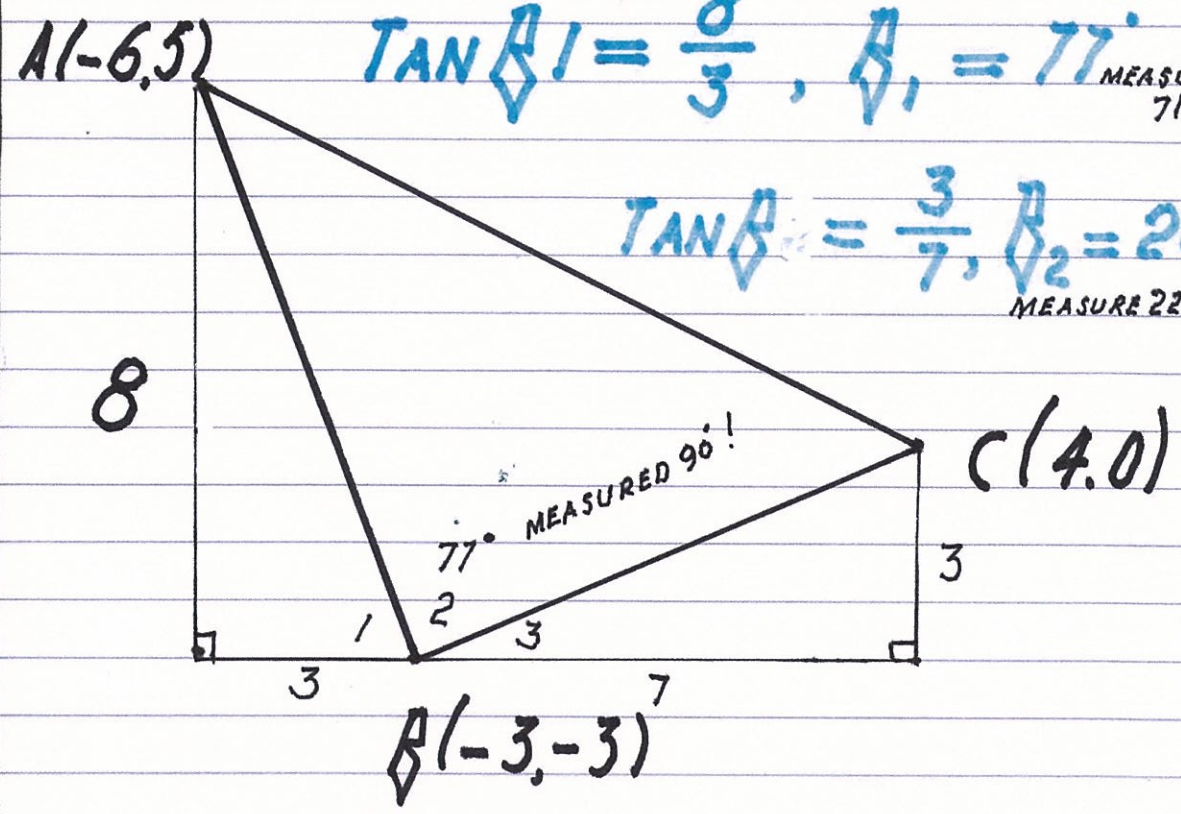
COS.<1 COS(y) START WITH THE SHORTER SIDE

$$\cos\theta = \frac{x-3}{4}$$

$$\sin\theta = \frac{7-y}{3}$$

$$\frac{(x-3)^2}{16} + \frac{(y-7)^2}{9} = 1$$

212 **ONLY DRAW WHAT YOU NEED**



$a = 8 \times \tan 63$	$c = 8 \cos 63$
$x = \sqrt{360 - 320 \cos 70}$	$A = 80 \sin 70 \ cm^2$ 213

214 $$\cos x = \frac{13^2 + 17^2 - 15^2}{2 \times 13 \times 17}$$

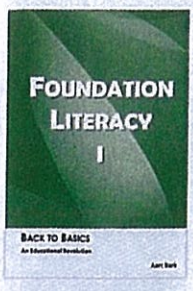
216 $$\frac{11}{\sin 102} = \frac{x}{\sin 40}$$

$$\frac{9}{\sin 65} = \frac{x}{\sin 75}$$

$$\frac{6}{\sin 40} = \frac{10}{\sin x}$$

217 **INVENT YOUR OWN VALUES**

CALCULATE x & COMPARE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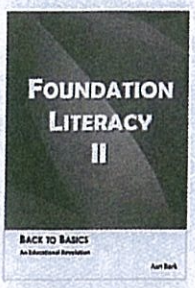
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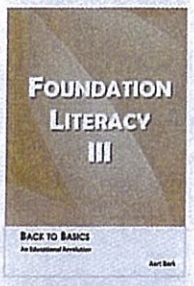
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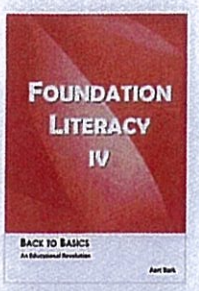
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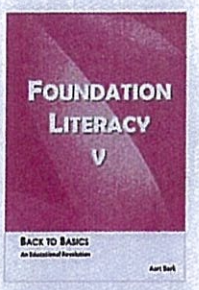
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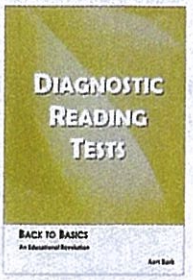
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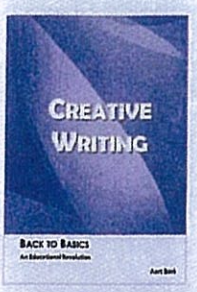
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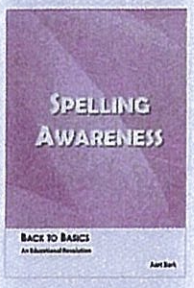
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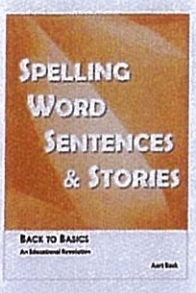
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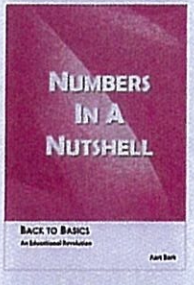
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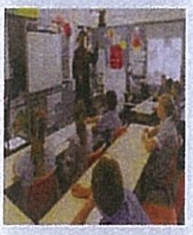
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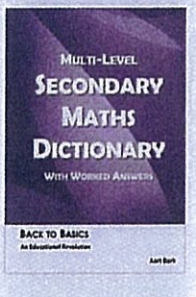
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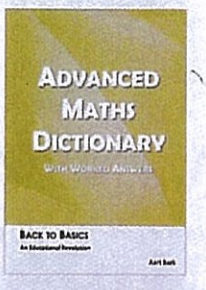
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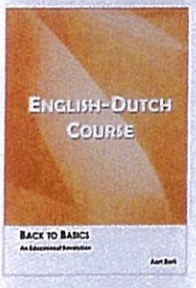
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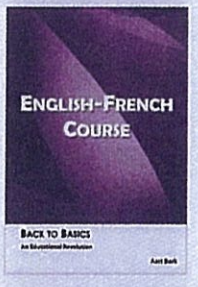
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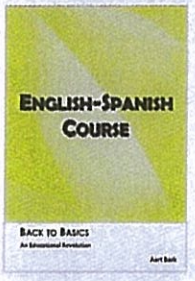
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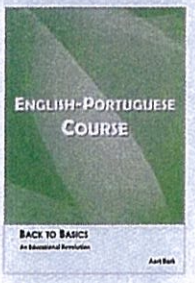
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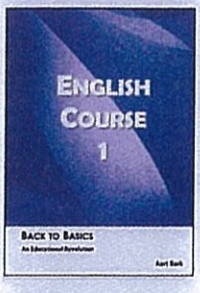
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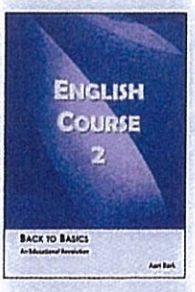
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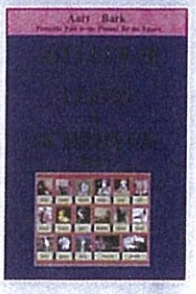
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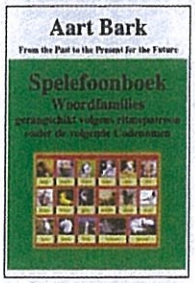
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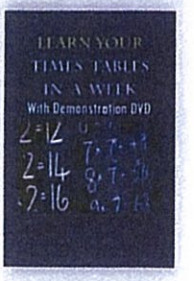
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